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Filtered Historical Simulation in VAR Models

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Abstract Historical simulation is becoming a widely used methodology for modeling financial risks of the Capital market. It is one of the standard methodologies for Value at Risk models. Major exchanges, such as Chicago Mercantile Exchange, started to adapt historical simulation methodology as marginal models instead of stress testing models and the parametric models. Among different approaches of the historical simulations, the filtered historical approach is one of the most common approaches. The approach aims to reflect the stylized facts of the behavior of the markets, such as the correlation of the realized volatility and the underlying. In this work, we will revisit the statistical evidence of the relationship of volatility and the underlying. We will assess the impact of the scaling on model outputs. This research offers two new aspects, a different approach of defining the scaling, and an illustration of the effect of the scaling under different market conditions.

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1. Introduction

The Value at Risk (VAR) of the historical simulation sets the VAR as the PnL loss at certain threshold percentile of a chosen historical period. One of the criticisms is that the historical simulation fails to recognize the current market condition and fails to adjust for volatility clustering. A widely adapted improvement of historical simulation is the so-called volatility scaling, where the simulation of the next day return is adjusted based on the volatility of the (immediate) past. The (immediate) past volatility is used as a measurement of the market conditions.

There are extensive literatures on volatility scaling. See [1] for a survey of approaches used in margin models. Detailed description of the method and the effect of the model on Various underlying are in [2]. Alternative methodologies are offered in [4], [5], [6], [7]. One of the impacts of the scaling is magnifying the return volatility during stress periods, thus increasing the possibility of the large moves, which leads to increase of the VAR. In this work, we will illustrate the effect of the volatility scaling during two types of markets, a horizontal market, where market moves horizontally without a big trend, and an easing market, where the market is coming out of a significant drawdown and recovering.

To measure quality of the VAR model, we will use the usual back-testing as the primary tool. We will count the breaches, compute the coverage ratio and the overshoots. As a secondary measure, we will look at the sum of the overshoots, which is the total market risk exposure of the central clearing party to its clients. We will also look at the stability of the VAR itself. The stability of the margin requirement is one of the desired properties of the VAR / Margin models as well.

We will use the QQQ (NASDAQ 100 ETF) as an example to explore the behavior of the scaling approaches. It appears that the market cycle has a big impact on the effectiveness of the scaling approach. The example shows that, the volatility scaling at the timing of an easing market, reduce the VAR, making the Var/Margin model aggressive.

This research aims to provide a practical consideration, in some sense, to optimize the choice of the length of lookback historical period. The length should depend on the characteristics of the underlying, such as the inherent volatility and the strengths of its mean reversion, and the current stages in the market cycle.

The contribution of the paper are two folds. First, the paper illustrates that the effectiveness of the filtering methodology depends on the current stage of the market cycle. Second, the paper proposes a new approach for scaling, which moderately improves the performance of the model on the data set studied.

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In the detailed discussion in sections below, we will see that in general, the length of the lookback period determines the balance / trade-offs between the stability and responsiveness of the model. Shorter lookback period means less stability and more responsiveness, and vice versa. The "right" balance should be made based on the business objective, and behavior of the underlying asset.

We will also see that the effect of the filtering on the VaR result depends of the current market cycles. In the recovering / uptrend market, the filtering reduces the VaR, and makes the model worse. In the flat market, the filtering will improve the model and reduces breaches slightly. In the downturn market, while no example given this research, the filtering will increase the responsiveness and provide a better model outcome, comparing to the HS model.

2. The Scaled Historical VAR Model

The Value at Risk (VAR) is the dollar amount of the potential loss within a particular period and a certain confidence interval. For simplicity, we will set the period to be one day and the confidence level at alpha equals 5%.

The unfiltered (adjusted) historical VAR for $1 portfolio of the underlining, based on the historical return of the underlying r(n), 1<=n<=N, where N is the size of the historical simulation / lookback period, is the percentile of the confidence interval. HS_VAR = - Percentile (r(i), alpha). For convenience, we will use positive sign for VAR. In this calculation, HS_VAR is the amount, as if any of the potential returns in the historical simulation period, could happen the next day.

This assumption is not consistent with some of the well understood facts of the capital market. For example, data shows that correlation between the STDEV of prior 21 days with the absolute value (ABS) of daily return of the underlying. When the volatility is high in the market, The next day move is likely higher. Correlation of ABS of next day return and STDEV (N), for N is 1, 5, 20, 60, 120, 250, 500 are as follows

Table 1, Correlation of ABS (next day return) and STDEV (N days)

1 day	5 days	20 days	60 days	120 days	250 days	500 days
-2.46%	26.10%	30.17%	31.19%	30.42%	17.64%	-5.04%

This is consistent with the market intuition. The hangover of the shock event could last for a week to half a year but less so for a year or 2 years.

To adjust for the volatility, the filtering approach is purposed in the Method 1 below in the literature. From the time series of the return, the Devol process divides the return by the volatility, measured with the standard deviation (STDEV) of immediate past N days. The Devol process intends to remove the effect of the realized volatility on the return. The Revol process multiplies the time series by volatility, so that the return is related to the realized volatility. The volatility of Devol and the Revol need not to be using the same time lookback periods. The time periods should be dependent on the underlying, following a calibration process.

Two ways to Devol and Revol are as follows. Method 1 is the common approach and Method 2 is proposed here for the first time in the literature. At time j, the VAR at j is

Method 1

VAR(j) = Revol(j) * Percentile (Return(i) / Devol(i), i<j)

Method 2

Revol(j) = Percentile (Return(i), i<j) / Devol(j).

Note that, in Method 2, if the Devol and Revol have the same time length, the scalar is the constant 1, and VAR is the same as the Historical Simulation (HS) method.

In the next a few sections, we will compare the two methods on different stages of the market cycle.

We will set the Devol and Revol Lookback periods to be 5 days, 20 days, 60 days, and 120 days. As shown in table 1, the correlation of the next day return with (immediate) past volatility are strongest at around 30%, for those full-time periods, 5 days, 20 days, 60 days, and 120 days. They are roughly a week, a month, a quarter, and a half a year. The correlation decreases when the period approach one year or longer.

The most common performance measure for VAR model and on margin models is the bridge ratio, which is defined as the breach count divided by the exposure date count (length of the backtesting data). For the ease of the capital management, it is also desirable that VAR or margin amount are relatively stable. Unpredictable margin requirements could make the trader's job much more difficult and reduce competitiveness of clearing houses. In the meantime, the margin requirement needs to reflect changing market conditions and reduce the breach ratio as its primary goal.

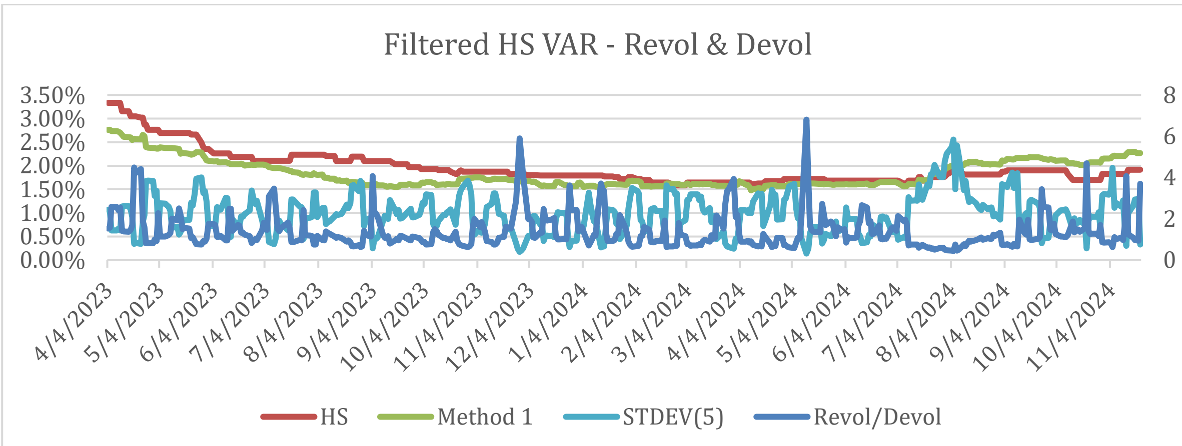
3. Revol and Devol with the same time periods

In the Graph 1 below, the right axis is for the ratio Revol(j) / Devol(j), for date j from 4/4/2023 to 11/4/2024. In the case where Devol and Revol are based on the same time periods, the ratio Revol(j)/Devol(j) is a constant 1. The Method 2 scaling approach is the same as the historical simulation (HS). In Graph 1 Method 2 result is omitted.

In Graph 1, left axis is for the HS_VAR, Method 1 (VAR) and STDEV (5) which is the STDEV of the rolling 5-day return. STDEV (5) is added in the graph as an indicator of market condition at times j. When the market is in turbulence, usually STDEV (5) jumps.

HS is calculated based on the historical simulation period of one year. We see that it is a relatively stable, that when the market moves and STDEV (5) jumps, the HS does not react to the market conditions well. We split the period to Period One from 4/4/2023 to 1/29/2024 and Period Two from 1/29/2024 to 11/24/2024. In Period One, the HS_VAR decreases steadily, indicating the market turbulence is rolling off. The market enters to a relatively calm condition. STDEV (5) is low. In Period Two, the market is mostly flat where jumps did occur occasionally, say around 8/4/2024.

Table 2 below has the performance summary of the models. The Method One (VAR) is worse than HS in terms of breach count (24 versus 19). The overall STDEV of the VAR, Method one clearly has much more volatility, 0.72% versus 0.36%. If we look further, in the Period One, due to scaling, the minimum VAR is only 0.24%. As in the Period One, the STDEV (120) are still high and the STDEV has dropped. The performance of the Method One is better in the Period Two in terms of the overshoot (16 versus 12), in terms of the sum of overshoot as well, (-9.74% versus -2.84%).

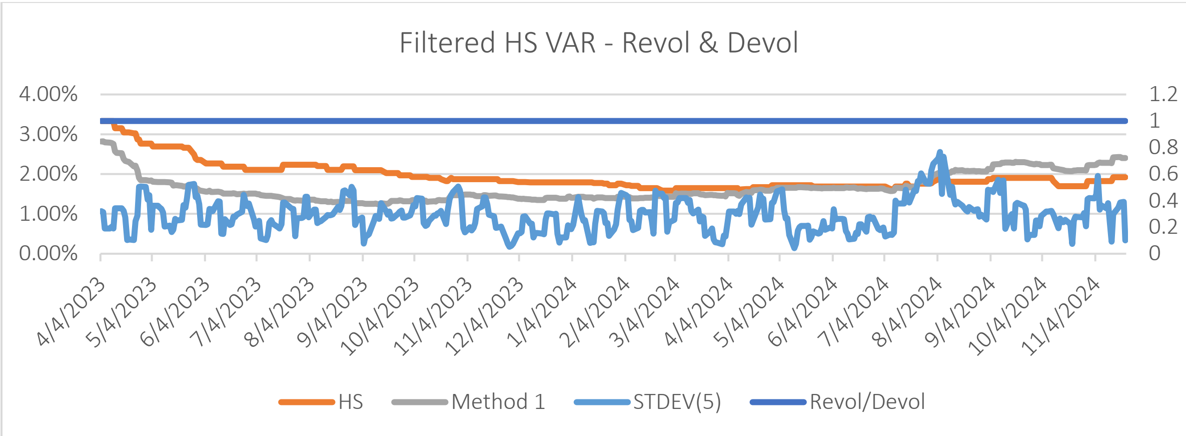


Graph 1, Filtered HS_VAR, Revol (5)/Devol (5)

Table 2, Performance Summary, Revol (5) / Devol (5)

	HS	Method 1			HS	Method 1
Breach Count	19	24		Time Period #1	4/4/2023	1/29/2024
Sum Overshoot	-10.35%	-5.43%		Breach Count	3	12
Data Count	428	428		Sum Overshoot	-0.61%	-2.58%
Breach %	4.44%	5.61%		Time Period #2	1/29/2024	11/21/2024
Max	3.33%	4.55%		Breach Count	16	12
Min	1.58%	0.24%		Sum Overshoot	-9.74%	-2.84%
STDEV	0.36%	0.72%				

Graph 2 and Table 3 below has the result of Revol = Devol = 120. In this case, the Method 1 (VAR) does not reflect the market change in weeks. While STDEV (5) moves around, the Method 1 (VAR) is little changed. The STDEV of Method 2 and HS are similar, 0.35% versus 0.36%. The overall model performance of Method 1 is worse than HS in the Period One with much higher breach count 15 versus 3. As the STDEV (120) is decreasing, the market becomes calm. In Method 1, the STDEV (120) in the denominator is inside the percentile calculation, and however the Revol STDEV (120) in the numerator is outside the percentile calculation, thus, the effect of scaling is the VAR is reduced during the period. Method 1 and HS have the similar performance in the Period Two. Overall the Method 1 is not an improvement of HS.



Graph 2, Filtered HS_VAR, Revol (120) / Devol (120)

Table 3, Performance Summary, Revol (120) / Devol (120)

	HS	Method 1			HS	Method 1
Breach Count	19	32		Time Period #1	4/4/2023	1/29/2024
Sum Overshoot	-10.35%	-15.30%		Breach Count	3	15
Data Count	428	428		Sum Overshoot	-0.61%	-6.41%
Breach %	4.44%	7.48%		Time Period #2	1/29/2024	11/21/2024
Max	3.33%	2.82%		Breach Count	16	17
Min	1.58%	1.24%		Sum Overshoot	-9.74%	-8.89%
STDEV	0.36%	0.35%				

4. Revol and Devol with longer Devol Period

In this section, we will discuss the general approach of the calibration of the Revol and the Devoll. To achieve a better model as discussed earlier, a better model should have less breaches and the VAR itself should be relatively stable. In the previous section we discussed the Revol and the Devol has the same length. In this section we will discuss what happens when the Devol is longer. The goal of the calibration is to find the balance between the reduction of the breach count / reflectiveness of the market change and the stability of the VAR itself. In next section, we will discuss when the Revol period is longer.

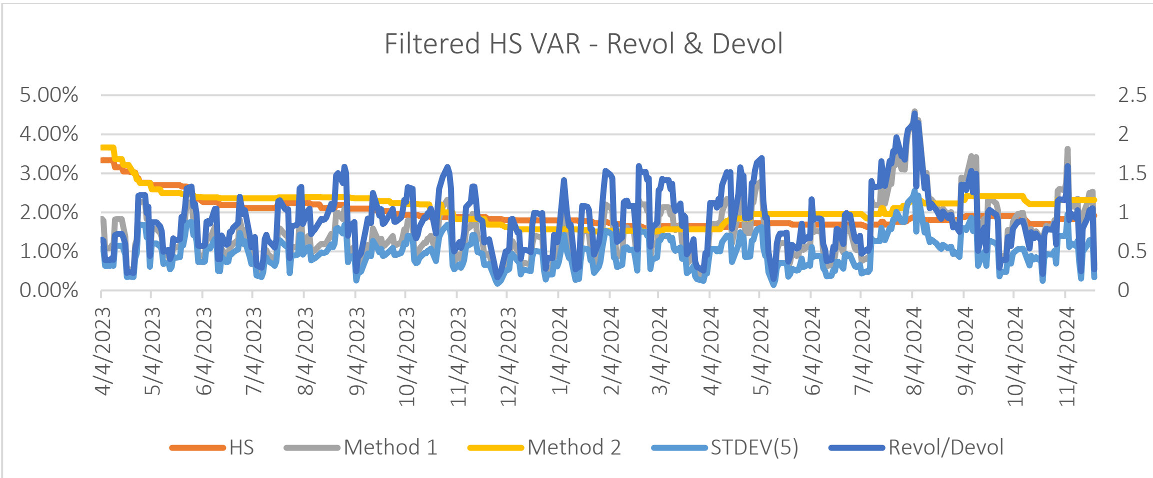
Calibration is to find the balance of the review reduction in the bridge count effectiveness of market change and the stability of VAR itself in the next section we will discuss when the Revol is longer.

Below is the result of Revol (5) / Devol (120) where the Devol is based on the 120-day STDEV and Revol is based on the 5-day STDEV. This choice where Devol is longer than the Revol, is widely accepted as the VAR is more reflective of the market change in the trend following way. As the market becomes calm, the VAR reduces. However, the performance of Method 1 is worse than the HS. The breach % is 8.4% for Method 1, which is higher than the target breach ratio. Also, the Revol is based on 5 days, the resulting VAR follows the market too closely, the STDEV of the VAR is high at the 0.73% while the STDEV of the HS_VAR is at 0.36%. As we discussed earlier, this high volatility for VAR is not desirable.

The Method 1 demonstrated different behaviors during two time periods. In the Period 1 where the market is calming down after a big turbulence, the Method 1 is worse than the HS. It produces a VAR as low as 0.24%. The scalar Revol (5) / Devol (120) dramatically reduce the overall level (/magnitude) of daily return, and its variability, as Revol (5) is much smaller than Devol (120). As a result, the breach count in the Period One is 18 while the breach count using HS is only three.

The Method 1 performs better in the Period Two than the HS method, consistent with the conclusion in the existing literature. The breach count is 11 (Method 1) versus 16 (HS). The sum of overshoot is (- 3.13%) (Method 1) versus (-9.74%) (HS).

Among three approaches, Method 2 seems to be the best. It has much lower breach count than Method 1. Comparing to HS, Method 2 has lower breach count than the HS in both time periods, 2 versus 3 in Period One, and 14 versus 16 in Period Two. The overall stability of VAR based on Method 2 measured with STDEV are similar, 0.42% (Method Two) versus 36% (HS).

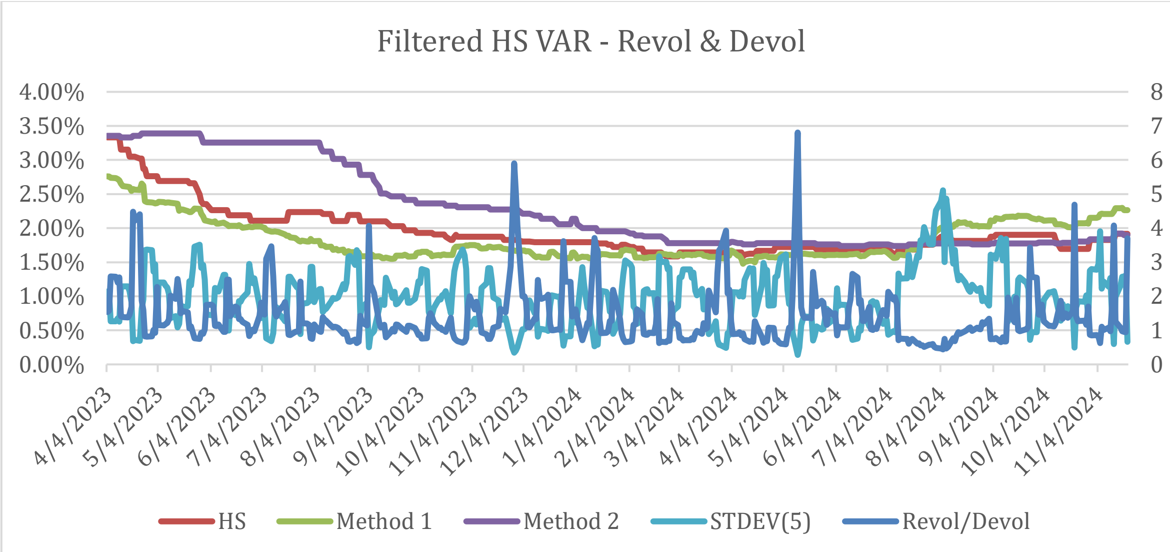


Graph 3, Filtered HS_VAR, Revol (5) / Devol (120)

Table 4, Result Summary, Revol (5) / Devol (120)

	HS	Method 1	Method 2
Breach Count	19	36	16
Sum Overshoot	-10.35%	-10.55%	-6.14%
Data Count	428	428	428
Breach %	4.44%	8.41%	3.74%
Max	3.33%	4.59%	3.66%
Min	1.58%	0.24%	1.51%
STDEV	0.36%	0.73%	0.42%
	HS	Method 1	Method 2
Time Period #1	4/4/2023	1/29/2024	
Breach Count	3	25	2
Sum Overshoot	-0.61%	-7.42%	-0.49%
Time Period #2	1/29/2024	11/21/2024	
Breach Count	16	11	14
Sum Overshoot	-9.74%	-3.13%	-5.65%

The results of Revol (20) / Devol (120) is shown in Graph 4 and Table 5 below. Overall, the result is as expected. The performance of the Method 1is the worst in Period One and the better in Period Two than the performance of HS. Compared to Revol of 5 days, due to the light increase of the lag in reflection of the market, the deficiency in time Period One and the benefit in time Period Two are dampened. Method 2 did not show improvement over HS.



Graph 4, Filtered HS_VAR, Revol (20) / Devol (120)

Table 5, Result Summary, Revol (20) / Devol (120)

	HS	Method 1	Method 2
Breach Count	19	30	22
Sum Overshoot	-10.35%	-13.62%	-10.34%
Data Count	428	428	428
Breach %	4.44%	7.01%	5.14%
Max	3.33%	3.51%	3.53%
Min	1.58%	0.78%	1.48%
STDEV	0.36%	0.54%	0.41%
	HS	Method 1	Method 2
Time Period #1	4/4/2023	1/29/2024	
Breach Count	3	17	5
Sum Overshoot	-0.61%	-7.75%	-0.82%
Time Period #2	1/29/2024	11/21/2024	
Breach Count	16	13	17
Sum Overshoot	-9.74%	-5.87%	-9.53%

5. Revol and Devel with longer Revol Period

What happens if Revol is 120 days and the Devol is at 5 days? As discussed earlier, if the goal is to have the VAR to be more reflective of the market condition, the Revol period should be shorter than the Devol period. If Revol is longer, then the VAR will have a negative correlation to the market condition when the market becomes calm with volatility reducing, the VAR (/ Margin) increases as the denominator for Devol decreases faster and sooner than the Revolnumerator. The resulting VAR are mean reverting. While some market participants hold this view, majority would not agree to the pattern intuitively.

If we look at the model performance in Period One and Two separately, this choice seems to compensate the deficiency of the Method 1 at the Period One. See the result below. During the Period One, the the Method 1 (VAR),

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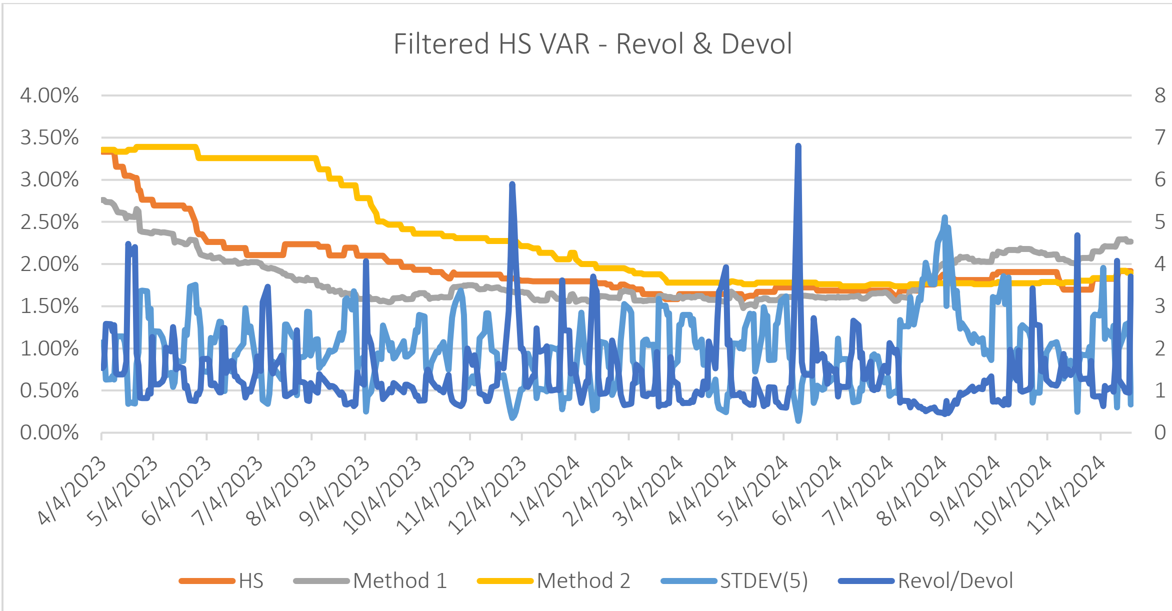
has a more gradual descent, which reduces the breach count during the Period One. The breach count is 8, much lower than the breach count of 17 in Revol (5) / Devol (120) case.

Looking at the Method 2, during Period O, the longer Revol Period Kept the VAR higher for longer. The breach count is reduced to 1 (Method 2) versus 3 (HS). If the conservatism of VAR is required depends on the behavior of the underlying, namely the strengths of the mean reversion. When the mean reversion of the underlying is weak, there might be a rationale to choose a longer Revol period.



Graph 5, Filtered HS_VAR, Revol (120) / Devol (5)

Table 6, Result Summary, Revol (120) / Devol (5)

	HS	Method 1	Method 2
Breach Count	19	25	14
Sum Overshoot	-10.35%	-11.57%	-9.60%
Data Count	428	428	428
Breach %	4.44%	5.84%	3.27%
Max	3.33%	2.76%	3.39%
Min	1.58%	1.48%	1.74%
STDEV	0.36%	0.29%	0.61%
	HS	Method 1	Method 2
Time Period #1	4/4/2023	1/29/2024	
Breach Count	3	8	1
Sum Overshoot	-0.61%	-2.78%	-0.11%
Time Period #2	1/29/2024	11/21/2024	
Breach Count	16	17	13
Sum Overshoot	-9.74%	-8.79%	-9.49%

6. Further research

The filtered historical simulation is one of the methodologies to incorporate some of the stylized fact in the capital market, such as the relationship between the return of underlying and the volatility. This research points out some of the performance and the behavior patterns under different market conditions. It appears that the impact of the filtered approach might be overstating the impact of those stylized facts. This might be one of the reasons for the conservativeness in the Central Counterparty Clearing House (CCP) margin models and high coverage ratio shown in CPMI-IOSCO quantitative disclosure. For example, see Chicago Mercantile Exchange (CME) disclosure [3]. The target coverage ratio is 99%, however the coverage ratio is higher than 99.9% consistently. As discussed in this paper, the conservativeness of the filtering depends on the current stages of the market cycle. If the adjustments to the filtering approach based on the market cycle are implemented, the CCP margin models might improve performance in the sense that they will remain effective and conservative, but the margin requirements would be reduced under certain market conditions.

An alternative conceptually sound approach is to simulate the next day distribution which is correlated to (immediate) prior volatility observed matching the observation. The approach should moderate some of the conservativeness in the filtered approach, reflecting the historical of durations more accurately than the outright "scaling" approach in the literature.

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