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Three-Component Piecewise-Linear Economic-Mathematical Mode and Method of Multivariate Prediction of Economic Process with Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

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Abstract

For the last 15 years in periodic literature there has appeared a series of scientific publications that has laid the foundation of a new scientific direction on creation of piecewise-linear economic-mathematical models at uncertainty conditions in finite dimensional vector space. Representation of economic processes in finite-dimensional vector space, in particular in Euclidean space, at uncertainty conditions in the form of mathematical models in connected with complexity of complete account of such important issues as: spatial in homogeneity of occurring economic processes, incomplete macro, micro and social-political information; time changeability of multifactor economic indices, their duration and their change rate. The above-listed one in mathematical plan reduces the solution of the given problem to creation of very complicated economic-mathematical models of nonlinear type. In this connection, it was established in these works that all possible economic processes considered with regard to uncertainty factor in finite-dimensional vector space should be explicitly determined in spatial-time aspect. Owing only to the stated principle of spatial-time certainty of economic process at uncertainty conditions in finite dimensional vector space it is possible to reveal systematically the dynamics and structure of the occurring process. In addition, imposing a series of softened additional conditions on the occurring economic process, it is possible to classify it in finite-dimensional vector space and also to suggest a new science-based method of multivariate prediction of economic process and its control in finite-dimensional vector space at uncertainty conditions, in particular, with regard to unaccounted factors influence.

Keywords: Finite-dimensional vector space; Unaccounted factors; Unaccounted parameters influence function; Principle of certainty of economic process in fine-dimensional space; Multi alternative forecasting; Principle of spatial-time certainty of economic process at uncertainty conditions in fine-dimensional space; Piecewise-linear economic-mathematical models in view of the factor of uncertainty in finite-dimensional vector space; Piecewise-linear vector-function; 3-Dimensional Vector Space; 3-Component Piecewise-Linear Economic-Mathematical Model in 3-Dimensional Vector Space; Hyperbolic surface.

I. Introduction. Formulation of the problem

In publications [1-5,12] theory of construction of piecewise-linear economic mathematical models with regard to unaccounted factors influence in finite-dimensional vector space was developed. A method for predicting economic process and controlling it at uncertainty conditions, and a way for defining the economic process control function in m-dimensional vector space, were suggested. In addition to this we should note that no availability of precise definition of the notion "uncertainty" in economic processes, incomplete classification of display of this phenomenon, and also no availability of its precise and clear mathematical representation places the finding of the solution of problems of prediction of economic process and this control to the higher level by its complexity.

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Many-dimensionality and spatial in homogeneity of the occurring economic process, time changeability of multifactor economic indices and also their change velocity give additional complexity and uncertainty. Another complexity of the problem is connected with reliable construction of such a predicting vector equation in the consequent small volume $\Delta V_{n+1}(x_1, x_2,... x_m)$ of finite-dimensional vector-space that could sufficiently reflect the state of economic process in the subsequent step. In other words, now by means of the given statistical points (vectors) describing certain economic process in the preceding volume $V = \sum_{N=1}^{N} \Delta V_N(x_1, x_2,... x_m)$ of finite-dimensional vector space R_m one can construct a predicting vector equation $\vec{Z}_{n+1}(x_1, x_2,... x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1, x_2,... x_m)$ of finite-dimensional vector space. The goal of our investigation is to formulate the notion of uncertainty for one class of economical processes and also to find mathematical representation of the predicting function $\vec{Z}_{n+1}(x_1, x_2,... x_m)$ for the given class of processes depending on so-called unaccounted factors functions. In connection with what has been said, below we suggest a method for constructing a predicting vector equation $\vec{Z}_{n+1}(x_1, x_2,... x_m)$ in the subsequent small volume $\Delta V_{n+1}(x_1, x_2,... x_m)$ of finite-dimensional vector space [1-7, 14].

II. Materials and methods:

In these publications, the postulate spatial-time certainty of economic process at uncertainty conditions in finite-dimensional vector space" was suggested, the notion of piecewise-linear homogeneity of the occurring economic process at uncertainty conditions was introduced, and also a so called the unaccounted parameters influence function $\omega_n(\lambda_n^{k_n}, \alpha_{n-1,n})$ influencing on the preceding volume $V = \sum_{N=1}^{N} \Delta V_n$ of economic process was suggested.

$$\vec{z}_n = \vec{z}_1 \left\{ 1 + A \left[1 + \omega_n(\lambda_n, \alpha_{n-1,n}) + \sum_{i=2}^{n-1 \geq 2} \omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) \right] \right\} \tag{1}$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) = \lambda_i^{k_i} \cos \alpha_{i-1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i-1} - \vec{z}_{i-1}^{\,k_{i-1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1} - \vec{z}_{i-1}^{\,k_{i-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|} \cos \alpha_{i-1,i} \tag{2}$$

$$\mu_i = (\mu_{i-1} - \mu_{i-1}^{k_{i-1}}) \frac{(\vec{a}_i - \vec{z}_{i-2}^{\,k_{i-2}})(\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}})}{(\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}})^2}, \text{ for } \mu_{i-1} \geq \mu_{i-1}^{k_{i-1}} \tag{3}$$

$$\omega_n(\lambda_n, \alpha_{n-1,n}) = \lambda_n \cos \alpha_{n-1,n} =$$

$$= \frac{\mu_n}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{n-1} - \vec{z}_{n-1}^{\,k_{n-1}}\right| \left|\vec{a}_{n+1} - \vec{z}_{n-1}^{\,k_{n-1}}\right|}{\vec{z}_1(\vec{z}_{n-1} - \vec{z}_{n-1}^{\,k_{n-1}})} \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|} \cos \alpha_{n-1,n} \tag{3.1}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)} \tag{4}$$

$$\mu_n = (\mu_{n-1} - \mu_{n-1}^{k_{n-1}}) \frac{(\vec{a}_n - \vec{z}_{n-2}^{\,k_{n-2}})(\vec{a}_{n+1} - \vec{z}_{n-1}^{\,k_{n-1}})}{(\vec{a}_{n+1} - \vec{z}_{n-1}^{\,k_{n-1}})^2}, \quad \mu_{n-1} \geq \mu_{n-1}^{k_{n-1}} \tag{5}$$

On this basis, it was suggested the dependence of the n-th piecewise-linear function $\vec{z}_n$ on the first piecewise-linear function $\vec{z}_1$ and all spatial type unaccounted parameters influence function $\omega_n(\lambda_n^{k_n}, \alpha_{n-1,n})$ influencing on the preceding interval of economic process, in the form Eqs. (1)–(5):

$$\vec{z}_n = \vec{z}_1 \left\{ 1 + A \left[1 + \omega_n(\lambda_n, \alpha_{n-1,n}) + \sum_{i=2}^{n-1\geq 2} \omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) \right] \right\} \quad (6)$$

where

$$\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) = \lambda_i^{k_i} \cos\alpha_{i-1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \cos\alpha_{i-1,i} \quad (7)$$

are unaccounted parameters influence functions influencing on the preceding ΔV_1, ΔV_2,.. ΔV_i small volumes of economic process;

$$\mu_i = (\mu_{i-1} - \mu_{i-1}^{k_{i-1}}) \frac{(\vec{a}_i - \vec{z}_{i-2}^{k_{i-2}})(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})}{(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})^2}, \quad \text{for } \mu_{i-1} \geq \mu_{i-1}^{k_{i-1}} \quad (8)$$

are arbitrary parameters referred to the i-th piecewise-linear straight line. And the parameters μ_i are connected with the parameter μ_{i-1} referred to the (i-1)-th piecewise-linear straight line, in the form Eq. (8);

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \quad (9)$$

is a constant quantity;

$$\omega_n(\lambda_n, \alpha_{n-1,n}) = \lambda_n \cos\alpha_{n-1,n} =$$

$$= \frac{\mu_n}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{n-1} - \vec{z}_{n-1}^{k_{n-1}}\right| \left|\vec{a}_{n+1} - \vec{z}_{n-1}^{k_{n-1}}\right|}{\vec{z}_1(\vec{z}_{n-1} - \vec{z}_{n-1}^{k_{n-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \cos\alpha_{n-1,n} \quad (10)$$

is the expression of the unaccounted parameters influence function that influences on subsequent small volume ΔV_N of finite-dimensional vector space. And the parameter μ_n referred to the n- piecewise-linear straight line is of the form:

$$\mu_n = (\mu_{n-1} - \mu_{n-1}^{k_{n-1}}) \frac{(\vec{a}_n - \vec{z}_{n-2}^{k_{n-2}})(\vec{a}_{n+1} - \vec{z}_{n-1}^{k_{n-1}})}{(\vec{a}_{n+1} - \vec{z}_{n-1}^{k_{n-1}})^2}, \quad \mu_{n-1} \geq \mu_{n-1}^{k_{n-1}} \quad (11)$$

Here the parameter μ_n is connected with the parameter μ_{n-1} of the preceding (n-1)-th piecewise-linear vector equation of the straightline in the form Eq. (11).

Thus, in finite-dimensional vector space, the system of statistical points (vectors) is represented in the vector form in the form of N piecewise-linear straight lines depending on the vector function of the first piecewise-linear straight-line $\vec{z}_1 = \lambda_1\vec{a}_1 + \mu_1\vec{a}_2$, and also on the unaccounted parameters influence function $\omega_n(\lambda_n, \alpha_{n-1,n})$ in all the investigated preceding volume of finite-dimensional vector space R_m .After that, in publications [6-11,13-15] a solution was found of solve a problem on prediction of economic process and its control at uncertainty conditions in finite-dimensional vector space.

It became clear, that the unaccounted parameters influence functions $\omega_n(\lambda_n, \alpha_{n-1,n})$ are integral characteristics of influencing external factors occurring in environment that are not a priori situated in functional chain of sequence of the structured model but render very strong functional influence both on the function and on the results of prediction quantities Eq. (6). It is impossible to fix such a cause by statistical means. This means that the investigated this or other economic process in finite dimensional vector space directly or obliquely is connected with many dimensionality and spatial inhomogenlity of the occurring economic process, with time changeability of multifactor economic indices, vector and their change velocity. This in its turn is connected with the fact that the used statistical data of economic process in finite-dimensional vector space are of inhomogeneous in coordinates and time unstationary events character.

We assume the given unaccounted factors functions $\omega_n(\lambda_n, \alpha_{n-1,n})$ hold on all the preceding interval of finite-dimensional vector space, the uncertainty character of this class of economic process. In such a statement, the problem on prediction of economic event on the subsequent small volume ΔV_{N+1} of finite-dimensional vector space will be directly connected in the first turn with the enumerated invisible external facts fixed on the earlier stages and their combinations, i.e., the functions $\omega_n(\lambda_n, \alpha_{n-1,n})$ that earlier hold in the preceding small volumes $\Delta V_1, \Delta V_2, \ldots\ldots \Delta V_N$ of finite-dimensional vector space. Therefore, by studying the problem on prediction of economic process on subsequent small volume ΔV_{N+1} it is necessary to be ready to possible influence of such factors.

In connection with such a statement of the problem, let's investigate behavior of economic process in subsequent small volume ΔV_{N+1} finite-dimensional vector space under the action of the desired unaccounted parameters function $\omega_n(\lambda_n, \alpha_{n-1,n})$ that was earlier fixed by us in preceding small volumes ΔV_n of finite-dimensional vector space, i.e., $\omega_2(\lambda_2, \alpha_{1,2})$, $\omega_3(\lambda_3, \alpha_{2,3})$, …., $\omega_N(\lambda_N, \alpha_{N-1,N})$.

In connection with what has been said, the problem on prediction of economic process and its control in finite-dimensional vector space may be solved by means of the introduced unaccounted parameters influence function $\omega_n(\lambda_n, \alpha_{n-1,n})$ in the following way.

Construct the (N+1)-th vector equation of piecewise-linear straight line $\vec{z}_{N+1} = \vec{z}_N^{\,k_N} + \mu_{N+1}(\vec{a}_{N+2} - \vec{z}_N^{\,k_N})$ depending on the vector equation of the first piecewise-linear straight line $\vec{z}_1$ and the desired unaccounted parameter influence function $\omega_\beta(\lambda_\beta, \alpha_{\beta-1,\beta})$ that we have seen in one of the preceding small volumes $\Delta V_1, \Delta V_2, \ldots\ldots \Delta V_N$ of finite-dimensional vector space. For that in Eqs. (6)–(11) we change the index n by $(N+1)$ and get:

$$\vec{z}_{N+1} = \vec{z}_1 \left\{ 1 + A \left[1 + \sum_{i=2}^{N} \omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) + \omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) \right] \right\} \tag{12}$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) = \lambda_i^{k_i} \cos \alpha_{i-1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i-1} - \vec{z}_{i-1}^{\,k_{i-1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1} - \vec{z}_{i-1}^{\,k_{i-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|} \cos \alpha_{i-1,i} \tag{13}$$

$$\mu_i = (\mu_{i-1} - \mu_{i-1}^{k_{i-1}}) \frac{(\vec{a}_i - \vec{z}_{i-2}^{\,k_{i-2}})(\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}})}{(\vec{a}_{i+1} - \vec{z}_{i-1}^{\,k_{i-1}})^2}, \qquad \mu_{i-1} \geq \mu_{i-1}^{k_{i-1}} \tag{14}$$

$$A = (\mu_1^{k_1} - \mu_1)\frac{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{15}$$

$$\omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) = \lambda_{N+1} cos\ \alpha_{N,N+1} =$$

$$= \frac{\mu_{N+1}}{\mu_1^{k_1} - \mu_1}\cdot\frac{\left|\vec{z}_N - \vec{z}_N^{k_N}\right|\left|\vec{a}_{N+2} - \vec{z}_N^{k_N}\right|}{\vec{z}_1(\vec{z}_N - \vec{z}_N^{k_N})}\cdot\frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\ \alpha_{N,N+1} \tag{16}$$

$$\mu_{N+1} = (\mu_N - \mu_N^{k_N})\frac{(\vec{a}_{N+1} - \vec{z}_{N-1}^{k_{N-1}})(\vec{a}_{N+2} - \vec{z}_N^{k_N})}{(\vec{a}_{N+2} - \vec{z}_N^{k_N})^2}, \mu_N \geq \mu_N^{k_N} \tag{17}$$

For the behavior of economic process on the subsequent small volume ΔV_{N+1} of finite-dimensional vector space to be as in one of the desired preceding ones in small volume ΔV_{β} it is necessary that the vector equations of piecewise-linear straight lines $\vec{z}_{N+1}$ and $\vec{z}_{\beta}$ to be situated in one of the planes of these vectors and to be parallel to one another, i.e.

$$\vec{z}_{N+1} = C\vec{z}_{\beta} \tag{18}$$

In connection with what has been said, to ΔV_{N+1} finite-dimensional space there should be chosen such a vector-point $\vec{a}_{N+2}$ that the piecewise-linear straight lines $\vec{z}_{N+1} = (\vec{a}_{N+2} - \vec{z}_N^{k_N})$ and $\vec{z}_{\beta} = (\vec{a}_{\beta+1} - \vec{z}_{\beta-1}^{k_{\beta-1}})$ could be situated in the same plane of these vectors and at the same time be parallel to each other (Fig. 1).

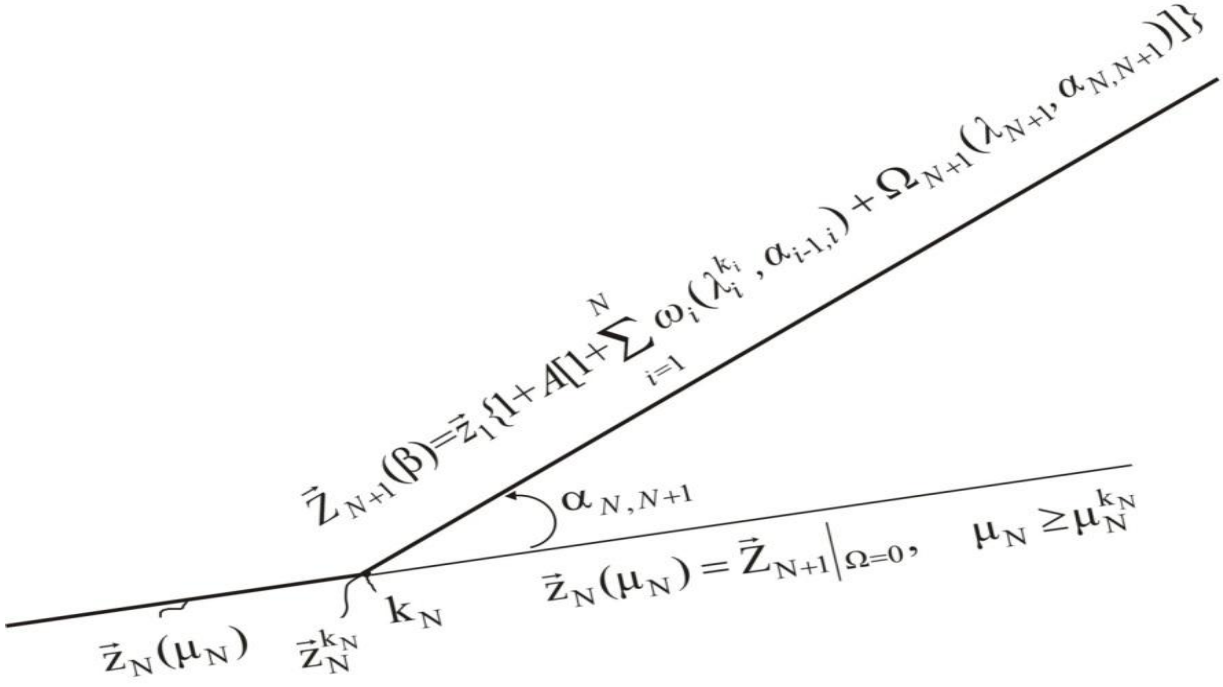

Fig.1. The scheme of construction of prediction function of economic process $\vec{Z}_{N+1}(\beta)$ at uncertainty conditions in finite-dimensional vector space R_m. Prediction function $\vec{Z}_{N+1}(\beta)$ will lie in the same plane with one of the desired preceding β -th piecewise-linear straight line and will be parallel to it.

In other words, they should satisfy the following parallelism condition:

$$(\vec{a}_{N+2} - \vec{z}_{N}^{\,k_N}) = C(\vec{a}_{\beta+1} - \vec{z}_{\beta-1}^{\,k_{\beta-1}}) \tag{19}$$

Here

$$\vec{a}_{N+2} = \sum_{m=1}^{M} a_{N+2,m}\vec{i}_m , \quad \vec{a}_{\beta+1} = \sum_{m=1}^{M} a_{\beta+1,m}\vec{i}_m ,$$

$$\vec{z}_{N}^{\,k_N} = \sum_{m=1}^{M} z_{N,m}^{k_N}\vec{i}_m , \quad \vec{z}_{\beta-1}^{\,k_{\beta-1}} = \sum_{m=1}^{M} z_{\beta-1,m}^{k_{\beta-1}}\vec{i}_m$$

Excluding in Eq. (19) the parameter C, we get:

$$\frac{a_{N+2,1} - z_{N,1}^{k_N}}{a_{\beta+1,1} - z_{\beta-1,1}^{k_{\beta-1}}} = \frac{a_{N+2,2} - z_{N,2}^{k_N}}{a_{\beta+1,2} - z_{\beta-1,2}^{k_{\beta-1}}} = \ldots\ldots = \frac{a_{N+2,M} - z_{N,M}^{k_N}}{a_{\beta+1,M} - z_{\beta-1,M}^{k_{\beta-1}}} \tag{20}$$

It is easy to define from system Eq. (20) the coefficients of the vector $\vec{a}_{N+2}$:

$$a_{N+2,2} = z_{N,2}^{k_N} + \frac{a_{\beta+1,2} - z_{\beta-1,2}^{k_{\beta-1}}}{a_{\beta+1,1} - z_{\beta-1,1}^{k_{\beta-1}}}(a_{N+2,1} - z_{N,1}^{k_N})$$

$$a_{N+2,3} = z_{N,3}^{k_N} + \frac{a_{\beta+1,3} - z_{\beta-1,3}^{k_{\beta-1}}}{a_{\beta+1,1} - z_{\beta-1,1}}(a_{N+2,1} - z_{N,1}^{k_N})$$

$$a_{N+2,M} = z_{N,M}^{k_N} + \frac{a_{\beta+1,M} - z_{\beta-1,M}^{k_{\beta-1}}}{a_{\beta+1,M-1} - z_{\beta-1,M-1}^{k_{\beta-1}}}(a_{N+2,M-1} - z_{N,M-1}^{k_N}) \tag{21}$$

In this case, the vector $\vec{a}_{N+2}$ will have the following final form:

$$\vec{a}_{N+2} = a_{N+2,1}\vec{i}_1 + a_{N+2,2}\vec{i}_2 + a_{N+2,3}\vec{i}_3 + \ldots\ldots + a_{N+2,M}\vec{i}_M \tag{22}$$

As the coordinates of the point (of the vector) $\vec{a}_{N+2}$ now are determined by means of the piecewise-linear vector $\vec{z}_{\beta} = \vec{a}_{\beta+1} - \vec{z}_{\beta-1}^{\,k_{\beta-1}}$ taken from one of the preceding stage of economic process, it is appropriate to denote them in the form $\vec{a}_{N+2}(\beta)$ [8-13]. This will show that the coordinates of the point $\vec{a}_{N+2}$ (3) were determined by means of piecewise-linear straight line $\vec{z}_{\beta}$. In this case it is appropriate to represent Eq. (22) in the following compact form:

$$\vec{a}_{N+2}(\beta) = \sum_{m=1}^{M} a_{N+2,m}(\beta)\vec{i}_m \tag{23}$$

Now, in the system of Eqs. (12)–(17), instead of the vector $\vec{a}_{N+1}$ we substitute the value of the vector $\vec{a}_{N+2}(\beta)$, and also instead of $\omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ introduce the denotation of the so-called predicting influence function with regard to unaccounted parameters $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$. In this case the prediction function of the economic process $\vec{Z}_{N+1}(\beta)$ with regard to influence of prediction function of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ will take the following form:

$$\vec{Z}_{N+1}(\beta) = \vec{z}_1 \left\{ 1 + A \left[1 + \sum_{i=2}^{N} \omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) + \Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) \right] \right\} \tag{24}$$

Here

$$\omega_i(\lambda_i^{k_i}, \alpha_{i-1,i}) = \lambda_i^{k_i} \cos \alpha_{i-1,i} =$$

$$= \frac{\mu_i^{k_i}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}}\right| \left|\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}}\right|}{\vec{z}_1(\vec{z}_{i-1} - \vec{z}_{i-1}^{k_{i-1}})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \cos \alpha_{i-1,i} \tag{25}$$

$$\mu_i = (\mu_{i-1} - \mu_{i-1}^{k_{i-1}}) \frac{(\vec{a}_i - \vec{z}_{i-2}^{k_{i-2}})(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})}{(\vec{a}_{i+1} - \vec{z}_{i-1}^{k_{i-1}})^2}, \qquad \mu_{i-1} \geq \mu_{i-1}^{k_{i-1}} \tag{26}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{27}$$

and the prediction function of influence of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ will take the form:

$$\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) = \lambda_{N+1} \cos \alpha_{N,N+1} \tag{28}$$

$$\lambda_{N+1} = \frac{\mu_{N+1}}{\mu_1^{k_1} - \mu_1} \cdot \frac{\left|\vec{z}_N - \vec{z}_N^{k_N}\right| \left|\vec{a}_{N+2}(\beta) - \vec{z}_N^{k_N}\right|}{\vec{z}_1(\vec{z}_N - \vec{z}_N^{k_N})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \tag{29}$$

$$\mu_{N+1} = (\mu_N - \mu_N^{k_N}) \frac{(\vec{a}_{N+1} - \vec{z}_{N-1}^{k_{N-1}})(\vec{a}_{N+2}(\beta) - \vec{z}_N^{k_N})}{(\vec{a}_{N+2}(\beta) - \vec{z}_N^{k_N})^2}, \mu_N \geq \mu_N^{k_N} \tag{30}$$

Here the vector $\vec{a}_{N+2}(\beta)$ is determined by Eq. (23).

Note the following points. It is seen from Eq. (11) that for $\mu_N = \mu_N^{k_N}$ the value of the parameter $\mu_{N+1} = 0$. By this fact from Eq. (28) it will follow that the value of the predicting function of influence of unaccounted parameters $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ will equal:

$$\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) = 0 \quad \text{for} \quad \mu_{N+1} = 0$$

$$\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) \neq 0 \quad \text{for} \quad \mu_{N+1} > 0 \tag{31}$$

This will mean that the initial point from which the (N+1)-th vector equation of the prediction function of economic process $\vec{Z}_{N+1}(\beta)$ will enanimate, will coincide with the final point of the n-th vector equation of piecewise-linear straight line $\vec{z}_N$ and equal:

$$Z_{N+1} = \vec{z}_1 \left\{ 1 + A \left[1 + \sum_{i=2}^{N} \omega_i (\lambda_i^{k_i}, \alpha_{i-1,i}) \right] \right\}, \qquad \text{for } \mu_{N+1} = 0 \tag{32}$$

For any other values of the parameter $\mu_{N+1} \neq 0$ the points of the $(N+1)$-th vector equation will be determined by Eq. (24).

It is seen from Eq. (28) that $\lambda_{N+1} = 0$ and $\Omega_{N+1}(\lambda_{N+1} = 0; \alpha_{N,N+1}) = 0$ will follow $\cos \alpha_{N,N+1} = 0$ and $\mu_{N+1} \neq 0$. This will correspond to the case when the influence of external unaccounted factors on subsequent small volume ΔV_{N+1} are as in the preceding small volume ΔV_N of finite-dimensional vector space. In this case it suffices to continue the preceding vector equation $\vec{z}_N$ to the desired point $\mu_{N+1} = \mu^*_{N+1} > \mu_N^{k_N}$ of subsequent small volume of finite-dimensional vector space. The value of the vector function $\vec{Z}_{N+1}(\mu^*_{N+1}) = \vec{z}_N(\mu^*_{N+1}; \lambda_N, \alpha_{N-1,N})$ at the point $\mu_{N+1} = \mu^*_{N+1}$ will be one of the desired prediction values of economic process in subsequent small volume ΔV_{N+1}. In this case, the value of the controlled parameter of unaccounted factors will be equal to zero, i.e.,

$$\Omega_{N+1}(\mu_{N+1} \neq 0; \lambda_{N+1} \neq 0; \cos \alpha_{N, N+1} = 0; \alpha_{N,N+1} = 0) = 0$$

For any other value of the parameter μ_{N+1}, taken on the interval $0 \leq \mu_{N+1} \leq \mu^*_{N+1}$ and $\cos \alpha_{N,N+1} \neq 0$, the corresponding prediction function of unaccounted parameters will differ from zero, i.e., $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1}) \neq 0$. Thus, choosing by desire the numerical values of unaccounted parameters function $\omega_\beta(\mu_{N+1}; \lambda_\beta, \alpha_{\beta-1,\beta}) = \Omega_{N+1}(\lambda^*_{N+1}, \alpha_{N, N+1})$ corresponding to preceding small volumes $\Delta V_1, \Delta V_2, \ldots. \Delta V_N$ and influencing by them beginning with the point $\mu_{N+1} = 0$ to the desired point μ^*_{N+1}, we get numerical values of predicting economic event $\vec{Z}_{N+1}(\mu^*_{N+1}; \lambda^*_{N+1}, \alpha_{N,N+1})$ on subsequent step of the small volume ΔV_{N+1} (Fig. 2).

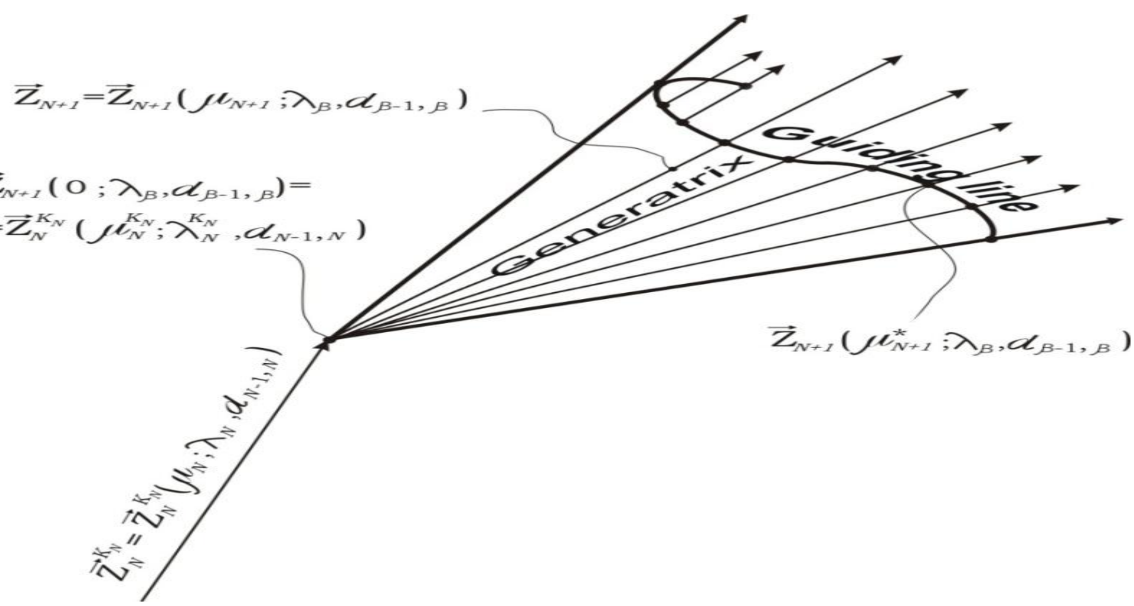

Fig.2. The graph of prediction of process and its control at uncertainty conditions in finite-dimensional vector space. It is represented in the form of hypersonic surface whose points, of directory will form the line of economic process prediction.

Taking into account the fact that by desire we can choose the predicting influence function of unaccounted parameters $\Omega^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$, then this function will represent a predicting control function of unaccounted factors, and its appropriate function $\vec{Z}^*_{N+1}(\mu^*_{N,N+1};\lambda^*_{N+1},\alpha_{N,N+1})$ will be a control aim function of economic event in finite-dimensional vector space. Speaking about unaccounted parameters prediction function $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ we should understand their preliminarily calculated values in previous small volumes $\Delta V_1, \Delta V_2, ...\ \Delta V_N$ of finite-dimensional vector space. Therefore, in Eq. (24) we used calculated ready values of the function $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$.

Thus, influencing by the unaccounted parameters influence functions of the form $\Omega_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ or by their combinations from the end of the vector equation of piecewise-linear straight line $\vec{z}^{k_N}_N(\mu^{k_N}_N;\ \lambda^{k_N}_N, \alpha_{N\text{-}1,N})$ situated on the boundary of the small volume $\vec{Z}_{N+1}(\beta) = \vec{Z}_{N+1}(\mu_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ there will originate the vectors ΔV_N and ΔV_{N+1}, lying on the subsequent small volume ΔV_{N+1}.

These vectors will represent the generators of hyperbolic surface of finite-dimensional vector space. The values of this series vector-functions for small values of the parameter $\mu_{N+1} = \mu^*_{N+1}$, i.e., $\vec{Z}_{N+1}(\mu^*_{N+1};\lambda_{N+1},\alpha_{N,N+1})$ will represent the points directrix of hypersonic surface of finite-dimensional vector space (Fig. 2). The series of the values of the points of directory hypersonic surface will create a domain of change of predictable values of the function of $\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$ in the further step in the small volume ΔV_{N+1}.

This predictable function will have minimum and maximum of its values $[\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})]_{\min}$ and $[\vec{Z}^*_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})]_{\max}$.

Thus, the found domain of change of predictable function of economic process in the form $\vec{Z}_{N+1}(\mu^*_{N+1};\lambda^*_{N+1},\alpha_{N,N+1})$, or in other words, the points of directrix of hyperbolic surface will represent the domain of economic process control in finite-dimensional vector-space.

III. 3-Component Piecewise-Linear Economic-Mathematical Model and Method of Multivariate Prediction of Economic Process With Regard to Unaccounted Factors Influence in 3-Dimensional Vector Space

In this article we give a number of practically important piecewise-linear economic-mathematical models with regard to unaccounted parameters influence factor in their-dimensional vector space. And by means of three-component piecewise-linear models suggest an appropriate method of multivariant prediction of economic process in subsequent stages and its control then at uncertainty conditions in 3-dimensional vector space [5-16]. In this section, we have given numerical construction of three-component piecewise-linear economic model with regard to unaccounted factors influence in 3-dimensional vector space, and construct appropriate vector-functions on the subsequent ΔV_4 small volume of 3-dimensional space.

Given a statistical table describing some economic process in the form of the points (vector) set $\{\vec{a}_n\}$ of 3-dimensional vector space R_3. Here the numbers a_{ni} are the coordinates of the vector $\vec{a}_n$ ($a_{n1}, a_{n2}, a_{n3}, \ldots\ldots a_{ni}$). With the help of the points (vectors) $\vec{a}_n$ represent the set of statistical points in the vector form in the form of 3-component piecewise-linear function [5-16]:

$$\vec{z}_1 = \vec{a}_1 + \mu_1(\vec{a}_2 - \vec{a}_1) \quad (33)$$

$$\vec{z}_2 = \vec{a}_2 + \mu_2(\vec{a}_3 - \vec{a}_2) \quad (34)$$

$$\vec{z}_3 = \vec{a}_3 + \mu_3(\vec{a}_4 - \vec{a}_3) \quad (35)$$

Here the vectors $\vec{z}_1 = \vec{z}_1(z_{11}, z_{12}, z_{13})$, $\vec{z}_2 = \vec{z}_2(z_{21}, z_{22}, z_{23})$ and $\vec{z}_3 = \vec{z}_3(z_{31}, z_{32}, z_{33})$ with the coordinates z_{ij} are given in the form of linear vector functions for the first, second and third piecewise-linear straight lines in 3-dimensional vector-space; the vectors (points) $\vec{a}_1(a_{11}, a_{12}, a_{13})$, $\vec{a}_2 = \vec{a}_2(a_{21}, a_{22}, a_{23})$, $\vec{a}_3 = \vec{a}_3(a_{31}, a_{32}, a_{33})$ and $\vec{a}_4 = \vec{a}_4(a_{41}, a_{42}, a_{43})$ are the given of 3-dimensional vector space R_3; $\mu_1 \geq 0$, $\mu_2 \geq 0$ and $\mu_3 \geq 0$ are arbitrary parameters of the first, second and third piecewise-linear straight lines. It holds the equality $\lambda_1 + \mu_1 = 1$, $\lambda_2 + \mu_2 = 1$ and $\lambda_3 + \mu_3 = 1$; $\alpha_{1,2}$ and $\alpha_{2,3}$ are the adjacent angles between the first and second and also between the second and third piecewise-linear straight lines; k_1 and k_2 are the intersection points between the piecewise-linear straight lines (Fig. 3).

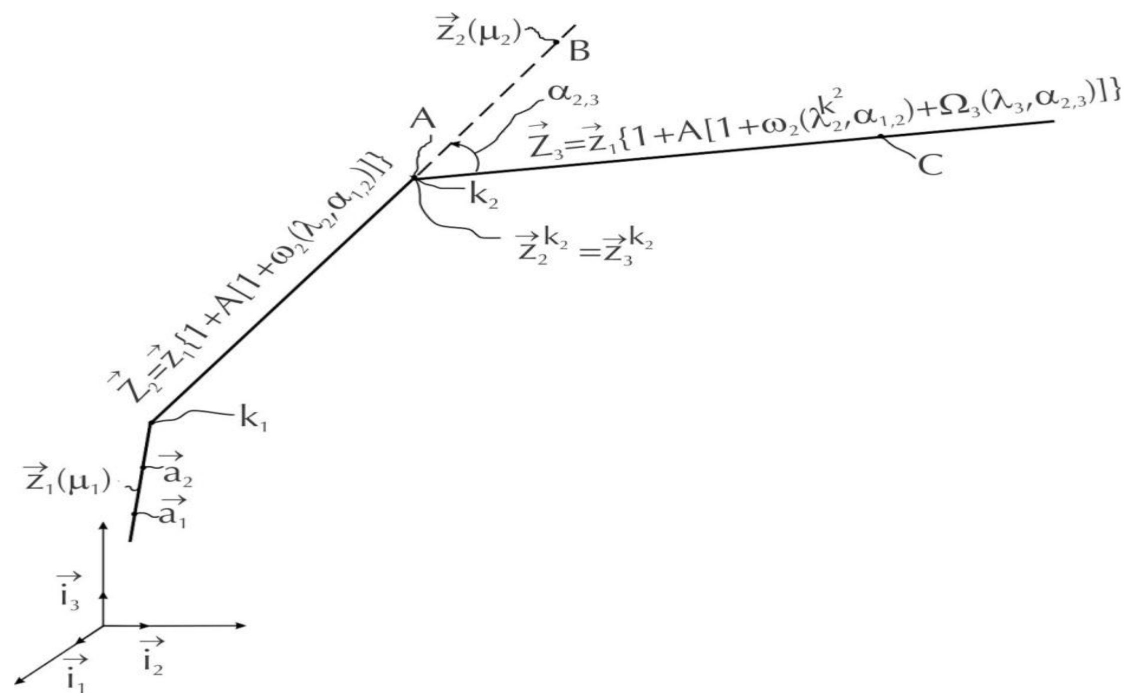

Fig. 3. Compact form of representation of numerical expression of the predicting vector function $\vec{Z}_3(\beta)$ constructed on the base of 2-component model n 3-dimensional vector space R_3.

As the intersection point between the second and third piecewise-linear straight lines in 3-dimensional vector space may also not coincide with the point $\vec{a}_3$, we denote this intersection point by $\vec{z}_2^{k_2}$ (Fig. 3). With regard to this factor, according to Eq. (1-11), an equation for the third piecewise-linear straight line is written in the form:

$$\vec{z}_3 = \vec{z}_2^{k_2} + \mu_3(\vec{a}_4 - \vec{z}_2^{k_2}) \tag{36}$$

Here $\vec{z}_2^{k_2}$ is the value of the point (vector) of the second piecewise-linear straight line at the k_2-th intersection point, represented by Eq. (1-11), and calculated for the value of the parameter $\mu_1 \geq \mu_1^{k_2}$, i.e., at the second intersection point k_2 and equal:

$$\vec{z}_2^{k_2} = \vec{z}_1^{k_2}[1 - A(\mu_1^{k_2})(1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2})]\} \tag{37}$$

where the parameter $\mu_1^{k_2}$ is calculated:

$$\mu_1^{k_2} = \mu_1^{k_1} + \mu_2^{k_2}\frac{(\vec{a}_3 - \vec{z}_1^{k_1})^2}{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)} \tag{38}$$

and the vector $\vec{z}_1^{k_2}$ is calculated by means of Eq. (33) at the point $\mu_1 = \mu_1^{k_2}$ in the form:

$$\vec{z}_1^{\,k_2} = \vec{a}_1 + \mu_1^{k_2}(\vec{a}_2 - \vec{a}_1) \tag{39}$$

and the coefficients $A(\mu_1^{k_2})$ and $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ are calculated by Eqs. (1-11) at the point $\mu_1 = \mu_1^{k_2}$ in the form:

$$A(\mu_1^{k_2}) = (\mu_1^{k_1} - \mu_1^{k_2})\frac{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{\,k_1} - \vec{a}_1|}{\vec{z}_1(\mu_1^{k_2})(\vec{z}_1^{\,k_1} - \vec{a}_1)} \tag{40}$$

$$\lambda_2^{k_2} = \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1^{k_2}} \cdot \frac{|\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{\,k_1}||\vec{a}_3 - \vec{z}_1^{\,k_1}|}{\vec{z}_1(\mu_1^{k_2})(\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{\,k_1})} \frac{\vec{z}_1(\mu_1^{k_2})(\vec{z}_1^{\,k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{\,k_1} - \vec{a}_1|} \tag{41}$$

$$cos\ \alpha_{1,2} = \frac{(\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{\,k_1})(\vec{a}_3 - \vec{z}_1^{\,k_1})}{|\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{\,k_1}| \cdot |\vec{a}_3 - \vec{z}_1^{\,k_1}|} \tag{42}$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2} cos\ \alpha_{1,2} \tag{43}$$

Here the value of the parameter at the second intersection point $\mu_2^{k_2}$ corresponding to the final point of the second piecewise-linear straight line is connected with the appropriate value of the parameter $\mu_1^{k_2}$ acting on the first straight line in the form Eq. (38):

$$\mu_2^{k_2} = (\mu_1^{k_2} - \mu_1^{k_1})\frac{(\vec{a}_3 - \vec{z}_1^{\,k_1})(\vec{a}_2 - \vec{a}_1)}{(\vec{a}_3 - \vec{z}_1^{\,k_1})^2} \tag{44}$$

Thus, giving the values of the parameter $\mu_1^{k_1}$ and $\mu_2^{k_2}$ at the intersection point k_1 and k_2 by Eq. (42) or Eq. (44), it is easy to define the appropriate value of the parameter $\mu_1^{k_2}$. Using [10,11,13(Capter 2),14,16] in 3-dimensional vector space write an equation for the points of the third piecewise-linear straight line depending on the vector equation of the first piecewise-linear straight line, spatial form of unaccounted parameters $\lambda_2^{k_2}$ and λ_3, and also on unaccounted parameters spatial influence functions $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$ in the form (Fig. 3):

$$\vec{z}_3 = \vec{z}_1\{1 + A\ [1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \omega_3(\lambda_3, \alpha_{2,3})]\}$$

for

$$\mu_1 \geq \mu_1^{k_2},\ \ \mu_2 \geq \mu_2^{k_2} \tag{45}$$

Here the unaccounted parameter influence function $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ is calculated by means of Eq. (11), the unaccounted parameter influence function $\omega_3(\lambda_3, \alpha_{2,3})$ is calculated in the form [10,11,13(Capter 2),14,16]:

$$\omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 \cdot cos\ \alpha_{2,3} \ \text{ for } \mu_3 \geq 0, \qquad \mu_2 \geq \mu_2^{k_2}$$

$$\lambda_3 = \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot \frac{|\vec{z}_2 - \vec{z}_2^{\,k_2}||\vec{a}_4 - \vec{z}_2^{\,k_2}|}{\vec{z}_1(\vec{z}_2 - \vec{z}_2^{\,k_2})} \cdot \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{|\vec{a}_2 - \vec{a}_1||\vec{z}_1^{\,k_1} - \vec{a}_1|} \quad \text{for } \mu_2 \geq \mu_2^{k_2},\ \ \mu_3 \geq 0 \tag{46}$$

$$cos\ \alpha_{2,3} = \frac{(\vec{z}_2(\mu_2) - \vec{z}_2^{\,k_2})(\vec{a}_4 - \vec{z}_2^{\,k_2})}{|\vec{z}_2(\mu_2) - \vec{z}_2^{\,k_2}||\vec{a}_4 - \vec{z}_2^{\,k_2}|},\ \text{for } \mu_2 \geq \mu_2^{k_2} \tag{47}$$

$$\mu_3 = (\mu_2 - \mu_2^{k_2}) \frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_4 - \vec{z}_2^{k_2})}{(\vec{a}_4 - \vec{z}_2^{k_2})^2} \text{ for } \mu_2 \geq \mu_2^{k_2} \tag{48}$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1}) \frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)}{(\vec{a}_3 - \vec{z}_1^{k_1})^2} \text{ for } \mu_2 \geq \mu_1^{k_1} \tag{49}$$

$$\mu_2^{k_2} = (\mu_1^{k_2} - \mu_1^{k_1}) \frac{(\vec{a}_3 - \vec{z}_1^{k_1})(\vec{a}_2 - \vec{a}_1)}{(\vec{a}_3 - \vec{z}_1^{k_1})^2} \text{ for } \mu_1^{k_2} \geq \mu_1^{k_1} \tag{50}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1 (\vec{z}_1^{k_1} - \vec{a}_1)} \tag{51}$$

Thus, giving the vectors (point) $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$, $\vec{z}_1^{k_1}$, $\vec{z}_2^{k_2}$ and $\vec{z}_2(\mu_2)$ Eq. (45) will represent a vector equation for the third piecewise-linear straight line $\vec{z}_3 = \vec{z}_3(\mu_1, \omega_3)$ in 3-dimensional vector space depending on the parameter $\mu_1 \geq \mu_1^{k_2}$ (i.e., for $\mu_2 \geq \mu_2^{k_2}$) and unaccounted parameters influence functions $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$ .Note that Eq. (45) defines all the points of the third piecewise-linear straight line in 3-dimensional space. To the case $\mu_3 = 0$ there will correspond the value of the initial point of the third straight line that will be expressed by the vector-function of the first piecewise-linear straight line $\vec{z}_1$, by the value of the parameters of intersection points of piecewise-linear straight lines $\mu_1^{k_1}$ and $\mu_2^{k_2}$, and also $Cos\ \alpha_{1,2}$ generated between the first and second piecewise-linear straight lines. It will equal:

$$\vec{z}_3\Big|_{\mu_3=0} = \vec{z}_1 \left\{ 1 + (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \cdot \left[1 + \right.\right.$$

$$\left.\left. + \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1} \frac{\left|\vec{z}_1 - \vec{z}_1^{k_1}\right| \left|\vec{a}_3 - \vec{z}_1^{k_1}\right|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})} \cdot \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|} cos\ \alpha_{1,2} \right]\right\} \tag{52}$$

Write the coordinate form of vector equation Eq. (45). Therefore, we have to take into account that in 3-dimensional vector space $\vec{z}_3 = \sum_{m=1}^{3} z_{3m} \vec{i}_m$ and $\vec{z}_1 = \sum_{m=1}^{3} z_{1m} \vec{i}_m$. In this case, the coordinates of the vector $\vec{z}_3$ i.e., z_{3m}, will be expressed by the coordinates of the first piecewise-linear straight line z_{1m}, spatial form of unaccounted parameters $\lambda_2^{k_2}$ and λ_3, and also on unaccounted parameters influence functions $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$ in the form:

$$z_3 = z_{1m}\{1 + A\ [1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \omega_3(\lambda_3, \alpha_{2,3})]\},\ (m = 1,2,3) \tag{53}$$

Here

$$A = (\mu_1^{k_1} - \mu_1) \frac{\sum_{i=1}^{3} (a_{2i} - a_{1i})^2}{\sum_{i=1}^{3} (a_{2i} - a_{1i})[a_1 + \mu_1(a_{2i} - a_{1i})]} \tag{54}$$

$$\lambda_2^{k_2} = \frac{\mu_2^{k_2}}{\mu_1^{k_1} - \mu_1^{k_2}} \cdot \frac{\sqrt{\sum_{i=1}^{3} \{a_{3i} - [a_{1i} + \mu_1^{k_1}(a_{2i} - a_{1i})]\}^2}}{\sqrt{\sum_{i=1}^{3} (a_{2i} - a_{1i})^2}} \tag{55}$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2} \cdot cos\ \alpha_{1,2} \tag{56}$$

$$\omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 \cdot cos\ \alpha_{2,3} = \\ = \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \cdot \frac{\sqrt{\sum_{i=1}^{3} (z_{2i} - z_{2i}^{k_2})^2} \cdot \sqrt{\sum_{i=1}^{3} (a_{4i} - z_{2i}^{k_2})^2}}{\sqrt{\sum_{i=1}^{3} (z_{2i} - z_{2i}^{k_2})}} \cdot \\ \cdot \frac{\sum_{i=1}^{3} z_{1i}(a_{2i} - a_{1i})}{\sum_{i=1}^{3} (a_{2i} - a_{1i})^2} \cdot cos\ \alpha_{2,3} \tag{57}$$

$$\mu_3 = (\mu_2 - \mu_2^{k_2}) \frac{\sum_{i=1}^{3} (a_{3i} - z_{1i}^{k_1})(a_{4i} - z_{2i}^{k_2})}{\sum_{i=1}^{3} (a_{4i} - z_{2i}^{k_2})^2}, \text{ for } \mu_2 \geq \mu_2^{k_2} \tag{58}$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1}) \frac{\sum_{i=1}^{3} (a_{3i} - z_{1i}^{k_1})(a_{2i} - a_{1i})}{\sum_{i=1}^{3} (a_{3i} - z_{1i}^{k_1})^2}, \text{ for } \mu_1 \geq \mu_1^{k_1} \tag{59}$$

$$\mu_2^{k_2} = (\mu_1^{k_2} - \mu_1^{k_1}) \frac{\sum_{i=1}^{3} (a_{3i} - z_{1i}^{k_1})(a_{2i} - a_{1i})}{\sum_{i=1}^{3} (a_{3i} - z_{1i}^{k_1})^2}, \text{ for } \mu_1^{k_2} \geq \mu_1^{k_1} \tag{60}$$

Now for the case of economic process represented in the form of three-component piecewise-linear economic-mathematical model investigate the prediction and control of such a process on the subsequent $\Delta V_4(x_1, x_2, x_3)$ small volume of 3-dimensional vector space with regard to unaccounted parameters influence functions $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3^{k_3}, \alpha_{2,3})$. The values of the unaccounted parameters functions $\omega_2(\lambda_2, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$ are unknown [5-16]. In [10,11,13(Capter 2),14,16], we developed a method for constructing an economic process predicting vector function $\vec{Z}_{N+1}(\beta)$ with regard to the introduced unaccounted parameter influence predicting function $\Omega_{N+1}(\lambda_{N+1}, \alpha_{N,N+1})$ in the m-th vector space that found its reflection in Eqs. (51)–(57). Apply this method to the case of the given three-component piecewise-linear economic process in 3-dimensional vector space. In this case, the predicting vector function $\vec{Z}_4(\beta)$ will be of the form (Fig. 4):

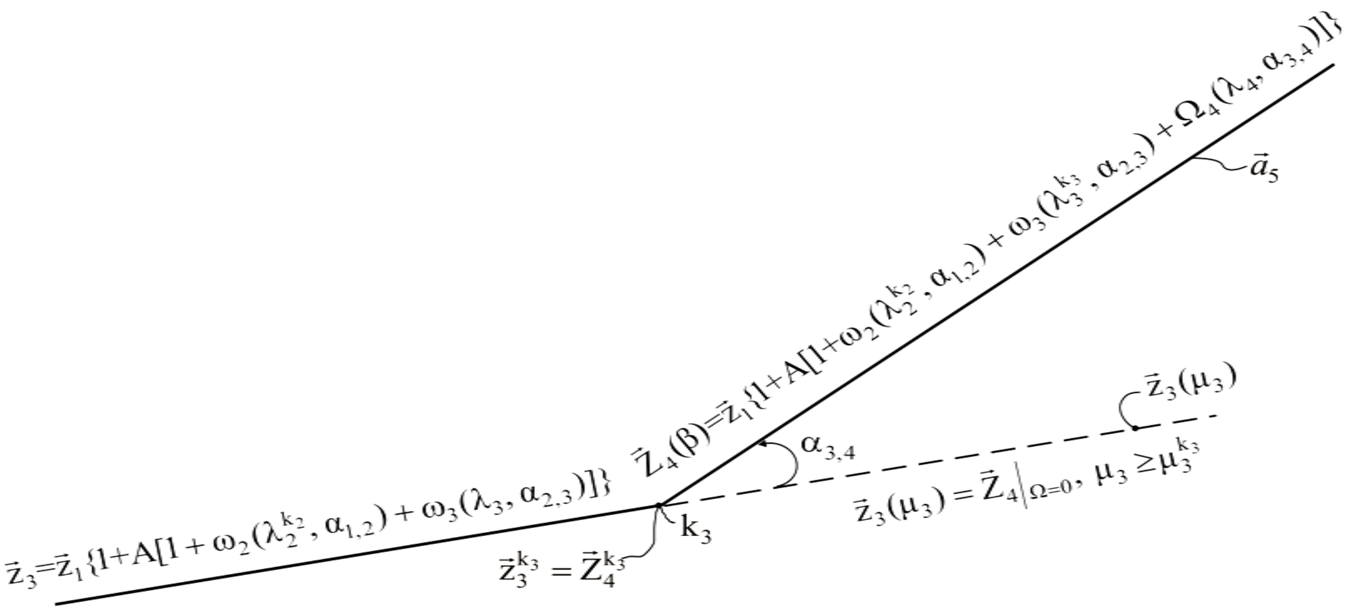

Fig.4. Construction of predicting vector functions $\vec{Z}_4(\beta)$ with regard to unaccounted parameter influence predicting function $\Omega_4(\lambda_4,\alpha_{3,4})$ on the base of 3-component piecewise-linear economic-mathematical model in 3-dimensional vector space R_3.

$$\vec{Z}_4(\beta)=\vec{z}_1\{1+A[1+\omega_2(\lambda_2^{k_2},\alpha_{1,2})+\omega_3(\lambda_3^{k_3},\alpha_{2,3})+\Omega_4(\lambda_4,\alpha_{3,4})]\}\quad \text{for}\ \ \beta=1,2 \tag{61}$$

where

$$\Omega_4(\lambda_4,\alpha_{3,4})=\lambda_4\cdot cos\ \alpha_{3,4} \tag{62}$$

$$\lambda_4=\frac{\mu_4}{\mu_1^{k_1}-\mu_1}\cdot\frac{\left|\vec{z}_3-\vec{z}_3^{\,k_3}\right|\cdot\left|\vec{a}_5(\beta)-\vec{z}_3^{\,k_3}\right|}{\vec{z}_1(\vec{z}_3-\vec{z}_3^{\,k_3})}\cdot\frac{\vec{z}_1(\vec{z}_1^{\,k_1}-\vec{a}_1)}{\left|\vec{a}_2-\vec{a}_1\right|\cdot\left|\vec{z}_1^{\,k_1}-\vec{a}_1\right|}$$

$$\text{for}\qquad \beta=1,2\ ,\ \mu_1\geq\ \ \mu_3^{k_3},\quad \mu_4\geq 0 \tag{63}$$

$$\mu_4=(\mu_3-\mu_3^{k_3})\cdot\frac{(\vec{a}_4-\vec{z}_2^{\,k_2})(\vec{a}_5(\beta)-\vec{z}_3^{\,k_3})}{(\vec{a}_5(\beta)-\vec{z}_3^{\,k_3})^2},\quad \mu_3\geq\mu_3^{k_3},\ \mu_4\geq 0\ \ \text{for}\ \ \beta=1,2 \tag{64}$$

The expressions of the unaccounted parameters functions $\omega_2(\lambda_2^{k_2},\alpha_{1,2})$ and $\omega_3(\lambda_3^{k_3},\alpha_{2,3})$ have the form Eq. (53)–(55) and

$$\omega_2(\lambda_2^{k_2},\alpha_{1,2})=\lambda_2^{k_2}\cdot cos\ \alpha_{1,2}=$$

$$=\frac{\mu_2^{k_2}}{\mu_1^{K_1}-\mu_1}\cdot\frac{\left|\vec{z}_1-\vec{z}_1^{\,k_1}\right|\cdot\left|\vec{a}_3-\vec{z}_1^{\,k_1}\right|}{\vec{z}_1(\vec{z}_1-\vec{z}_1^{\,k_1})}\cdot\frac{\vec{z}_1(\vec{z}_1^{\,k_1}-\vec{a}_1)}{\left|\vec{a}_2-\vec{a}_1\right|\cdot\left|\vec{z}_1^{\,k_1}-\vec{a}_1\right|}cos\ \alpha_{1,2} \tag{65}$$

$$\mu_2 = (\mu_1 - \mu_1^{k_1}) \cdot \frac{(\vec{a}_2 - \vec{a}_1)(\vec{a}_3 - \vec{z}_1^{k_1})}{(\vec{a}_3 - \vec{z}_1^{k_1})^2}, \qquad \mu_1 \geq \mu_1^{k_1} \tag{66}$$

$$A = (\mu_1^{k_1} - \mu_1) \cdot \frac{\left|\vec{a}_2 - \vec{a}_1\right| \cdot \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{67}$$

Here the vector $\vec{a}_5(\beta)$ for each value of $\beta = 1,2$, according [13(Capter 2), 15], is of the form:

$$\vec{a}_5(\beta) = a_{51}(\beta)\vec{i}_1 + a_{52}(\beta)\vec{i}_2 + a_{53}(\beta)\vec{i}_3 = \sum_{m=1}^{3} a_{5m}(\beta)\cdot \vec{i}_m \qquad \text{for } \beta = 1,2 \tag{68}$$

And by means of Eq. (37), the coordinates of the vector $\vec{a}(\beta)$ will be expressed by the coordinates of the vectors $\vec{a}_{\beta+1}$, $\vec{z}_{\beta-1}^{k_{\beta-1}}$ and $\vec{z}_3^{k_3}$ in the form:

$$C_\beta = \frac{a_{51}(\beta) - z_{31}^{k_3}}{a_{\beta+1,1} - z_{\beta-1,1}^{k_{\beta-1}}} = \frac{a_{52}(\beta) - z_{32}^{k_3}}{a_{\beta+1,2} - z_{\beta-1,2}^{k_{\beta-1}}} = \frac{a_{53}(\beta) - z_{33}^{k_3}}{a_{\beta+1,3} - z_{\beta-1,3}^{k_{\beta-1}}} \tag{69}$$

Hence, by Eq. (69), the coordinates $a_{52}(\beta)$ and $a_{53}(\beta)$ will be expressed by the arbitrarily given coordinate $a_{51}(\beta) > z_{31}^{k_3}$, in the form:

$$a_{52}(\beta) = z_{32}^{k_3} + (a_{51}(\beta) - z_{31}^{k_3})\frac{a_{\beta+1,2} - z_{\beta-1,2}^{k_{\beta-1}}}{a_{\beta+1,1} - z_{\beta-1,1}^{k_{\beta-1}}}$$

$$a_{53}(\beta) = z_{33}^{k_3} + (a_{51}(\beta) - z_{31}^{k_3})\frac{a_{\beta+1,3} - z_{\beta-1,3}^{k_{\beta-1}}}{a_{\beta+1,1} - z_{\beta-1,1}^{k_{\beta-1}}} \tag{70}$$

Here the coefficients $a_{\beta+1,m}$ and $z_{\beta-1,m}^{k_{\beta-1}}$ are the coordinates of the vectors $\vec{a}_{\beta-1}$ and $\vec{z}_{\beta-1}$ in 3-dimensional vector space and equal:

$$\vec{a}_{\beta-1} = \sum_{m=1}^{3} a_{\beta-1,m}\vec{i}_m, \quad \vec{z}_{\beta-1} = \sum_{m=1}^{3} z_{\beta-1,m}\vec{i}_m \tag{71}$$

Note that in the vectors $\vec{Z}_4(\beta)$ and $\vec{a}_5(\beta)$ the index (β) in the parenthesis means that the vector $\vec{Z}_4(\beta)$ is parallel to the β-th piecewise-linear vector-function $\vec{z}_\beta$. This will mean that the occurring economic process, beginning with the point $\vec{z}_3^{k_3}$ will occur by the scenario of the β-th piecewise-linear equation. In our example $\beta = 1,2$. In our case, there will be three predicting functions, i.e., $\vec{Z}_4(1)$, $\vec{Z}_4(2)$ and the case when the influence of unaccounted factors $\vec{Z}_4(0)$ will not be available. In all these cases, the predicting vector-functions $\vec{Z}_\beta(\beta)$ will emanate from one point $\vec{z}_3^{k_3}$, and the predicting vector-function $\vec{Z}_4(1)$ will be parallel to the first piecewise-linear straight line; $\vec{Z}_4(2)$ will be continuation of the third vector straight line $\vec{Z}_4(0)$, and all of them will emanate from one point $\vec{z}_3^{k_3}$. The expression $\cos\alpha_{3,4}$ corresponding to the cosine of the angle between the third piecewise-linear straight line $\vec{z}_3$ and the predicting fourth vector straight line $\vec{Z}_\beta(\beta)$ for each value of β on the base of scalar product of two vectors is represented in the form (Fig. 4):

$$cos\ \alpha_{3,4} = \frac{(\vec{z}_3 - \vec{z}_3^{\,k_3})(\vec{a}_5(\beta) - \vec{z}_3^{\,k_3})}{\left|\vec{z}_3 - \vec{z}_3^{\,k_3}\right|\left|\vec{a}_5(\beta) - \vec{z}_3^{\,k_3}\right|} \qquad (72)$$

IV. Results Method of Numerical Calculation of Three-Component Economic-Mathematical Model and Definition of Predicting Vector Function with Regard to Unaccounted Factors Influence in 3-Dmensional Vector Space

In this section, we have given numerical calculates of three-component piecewise-linear economic-mathematical model with regard to unaccounted parameters influence in 3-dimensional vector space, and construct appropriate predicting vector functions $\vec{Z}_4(1)$, $\vec{Z}_4(2)$ and $\vec{Z}_4(0)$ on subsequent ΔV_4 small volume of 3-dimensional space [5-16].

Consider the case of economic process given in the form of the statistical points (vectors) set $\{a_n\}$ in 3-dimensional vector space R_3 represented in the form of three-component piecewise-linear function of the form Eqs. (33)–(35). The vectors $\vec{a}_i = \vec{a}_i(a_{i1}, a_{i2}, a_{i3})$ (where $i = 1,2,3$) are the given points of 3-dimensional vector space R_3 and have the form:

$$\begin{aligned}
\vec{a}_1 &= \vec{i}_1 + \vec{i}_2 + \vec{i}_3, \\
\vec{a}_2 &= 3\vec{i}_1 + 2\vec{i}_2 + 4{,}5\vec{i}_3, \\
\vec{a}_3 &= 6\vec{i}_1 + 4\vec{i}_2 + 7\vec{i}_3, \\
\vec{a}_4 &= 8\vec{i}_1 + 9\vec{i}_2 + 10\ \vec{i}_3
\end{aligned} \qquad (73)$$

Below, by means of these vectors we have showed a method for calculating a chain form of each piecewise-linear vector equation depending on the first piecewise-linear vector straight line $\vec{z}_1$, cosines of the angles $cos\ \alpha_{1,2}$ and $cos\ \alpha_{2,3}$ generated between the adjacent first and second and also third and fourth piecewise-linear vector lines, and also on the parameter μ_1 corresponding to the first vector line [5-16].

Substituting Eq. (73) in Eq. (33), the equation of the first straight line in the coordinate form will be of the form:

$$\vec{z}_1 = (1 + 2\mu_1)\vec{i}_1 + (1 + \mu_1)\vec{i}_1 + (1 + 3{,}5\mu_1)\vec{i}_3 \qquad (74)$$

Giving the value of the parameter μ_1 for the intersection point k_1 between the first and second piecewise-linear straight lines of the form $\mu_1^{k_1} = 1{,}5$, the coordinate form of the intersection point $\vec{z}_1^{\,k_1}$ is defined from Eq. (74) in the form:

$$\vec{z}_1^{\,\mu_1^{k_1}} = \vec{z}_1^{\,1,5} = 4\vec{i}_1 + 2{,}5\vec{i}_1 + 6{,}25\ \vec{i}_3 \qquad (75)$$

By means of intersection point Eq. (75) and the given point $\vec{a}_3$ on the second straight line, by Eq. (49) set up a numerical relation between the parameters μ_1 and μ_2 in the form:

$$\mu_2 = 1{,}1927\ \ (\mu_1 - 1{,}5),\ \text{for}\ \mu_1 \geq 1{,}5,\ \mu_2 \geq 0 \qquad (76)$$

Hence:

$$\mu_1 = 1{,}5 + 0{,}8384\ \ \mu_2 \qquad (77)$$

Eq. (76) means that on the second piecewise-linear straight line, to the value of the parameter μ_2 there will be determined appropriate value of the operator μ_1 by Eq. (77). For the given value of the parameter $\mu_2^{k_2}$, corresponding to the intersection point between the second and third piecewise-linear straight lines equal 2, i.e., $\mu_2^{k_2} = 2$ from Eq. (77) or Eq. (50) the appropriate value of the parameter $\mu_1^{k_2}$ will equal:

$$\mu_1^{k_2} = 3{,}1768 \tag{78}$$

This means that when the parameter μ_2 corresponding to the points of the second piecewise-linear straight line will change within $0 \le \mu_2 \le 2$, then the appropriate value of the parameter μ_1 will change in the interval:

$$1{,}5 \le \mu_1 \le 3{,}1768 \tag{79}$$

This case will correspond to the case of the segment of the second straight line. For the value of the parameter $\mu_2 \ge 2$ the appropriate value of the parameter μ_1, will be $\mu_1 \ge 3{,}1768$. This case will correspond to the vector equation of the second straight line restricted from one end.

Now establish the form of the vector equation of the second piecewise-linear straight line depending on the vector equation of the first piecewise-linear straight line $\vec{z}_1$, $\cos\alpha_{1,2}$ and the parameter μ_1:

$$\vec{z}_2 = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2, \alpha_{1,2})]\} \tag{80}$$

where the coefficient A, the unaccounted factor parameter λ_2 and the unaccounted parameters function $\omega_2(\lambda_2, \alpha_{1,2})$ will be of the form:

$$A = (\mu_1^{k_1} - \mu_1)\frac{\left|\vec{z}_1^{k_1} - \vec{a}_1\right|\left|\vec{a}_2 - \vec{a}_1\right|}{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)} \tag{81}$$

$$\lambda_2 = \frac{\mu_2}{\mu_1^{k_1} - \mu_1}\frac{\left|\vec{z}_1 - \vec{z}_1^{k_1}\right|\left|\vec{a}_3 - \vec{z}_1^{k_1}\right|}{\vec{z}_1(\vec{z}_1 - \vec{z}_1^{k_1})}\frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{k_1} - \vec{a}_1\right|} \tag{82}$$

$$\omega_2(\lambda_2, \alpha_{1,2}) = \lambda_2 \cdot \cos\alpha_{1,2} \tag{83}$$

$$\cos\alpha_{1,2} = \frac{(\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{k_1})(\vec{a}_3 - \vec{z}_1^{k_1})}{\left|\vec{z}_1(\mu_1^{k_2}) - \vec{z}_1^{k_1}\right|\left|\vec{a}_3 - \vec{z}_1^{k_1}\right|} \tag{84}$$

Note that by Eq. (80) we must carry out numerical calculation for the values of the parameter μ_1. In conformity to our problem, we should use the range of the parameter μ_1 given in Eq. (79).

Determine the numerical values of the coefficients A, λ_2, $\cos\alpha_{1,2}$ and $\omega_2(\lambda_2, \alpha_{1,2,})$. For that substitute Eqs. (73–76) and the value of the parameter $\mu_1^{k_1} = \mu_1^{1,5} = 1{,}5$ in Eq. (81)–(84), and get:

$$A = -25{,}875 \cdot \varphi_0(\mu_1)\text{, for } 1{,}5 \le \mu_1 \le 3{,}1768 \tag{85}$$

$$\lambda_2 = 0{,}1203 \cdot \varphi_1(\mu_1)\text{, for } 1{,}5 \le \mu_1 \le 3{,}1768 \tag{86}$$

where

$$\varphi_0(\mu_1) = \frac{\mu_1 - 1{,}5}{9{,}75 + 25{,}875\ \mu_1} \tag{86a}$$

$$\varphi_1(\mu_1) = \frac{9,75 + 25,875\,\mu_1}{9,75 + 19,375\,\mu_1 - 17,25\,\mu_1^2}\sqrt{\begin{matrix}38,8125 - 51,75\,\mu_1 + \\ + 17,25\,\mu_1^2\end{matrix}} \quad (86b)$$

$$\cos\,\alpha_{1,2} = 0,7494 \quad (87)$$

$$\omega_2(\lambda_2, \alpha_{1,2}) = 0,0901 \cdot \varphi_1(\mu_1) \quad (88)$$

Substituting the numerical expressions of A, λ_2, $\cos\,\alpha_{1,2}$ and $\omega_2(\lambda_2, \alpha_{1,2,})$ Eqs. (85)–(88) in Eq. (80), find the final form of the vector function of the second piecewise-linear straight line depending on the first piecewise-linear straight line $\vec{z}_1$, and $\cos\,\alpha_{1,2}$ in the form:

$$\vec{z}_2 = \vec{z}_1 \cdot \varphi_2(\mu_1) \text{ for } 1,5 \le \mu_1 \le 3,1768 \quad (89)$$

where

$$\varphi_2(\mu_1) = 1 - 25,8751 \cdot \varphi_0(\mu_1) \cdot [1 + 0,0901 \cdot \varphi_1(\mu_1)] \quad (90)$$

Note that the obtained Eq. (89) is a vector equation of the second straight line where the value of the parameter $\mu_1 \ge 1,5$. When we impose on the parameter μ_1 the condition $1,5 \le \mu_1 \le 3,1768$, Eq. (89) will represent a vector equation of the second piecewise-linear segment.

Calculate the value of the intersection point of the second and third piecewise-linear straight lines, i.e., at the point κ_2. Therefore, according to approximation of piecewise-linear straight lines, for the intersection point accept the value of the parameter $\mu_2^{\kappa_2} = 2$, and the approximate value of the parameter $\mu_1^{\kappa_2}$ calculated earlier will correspond to the upper value of inequality Eq. (79), i.e., $\mu_1^{\kappa_2} = 3,1768$. In this case, we find the value of the vector function $\vec{z}_2^{k_2}$ in the coordinate form from Eq. (89) in the following form (Fig. 5):

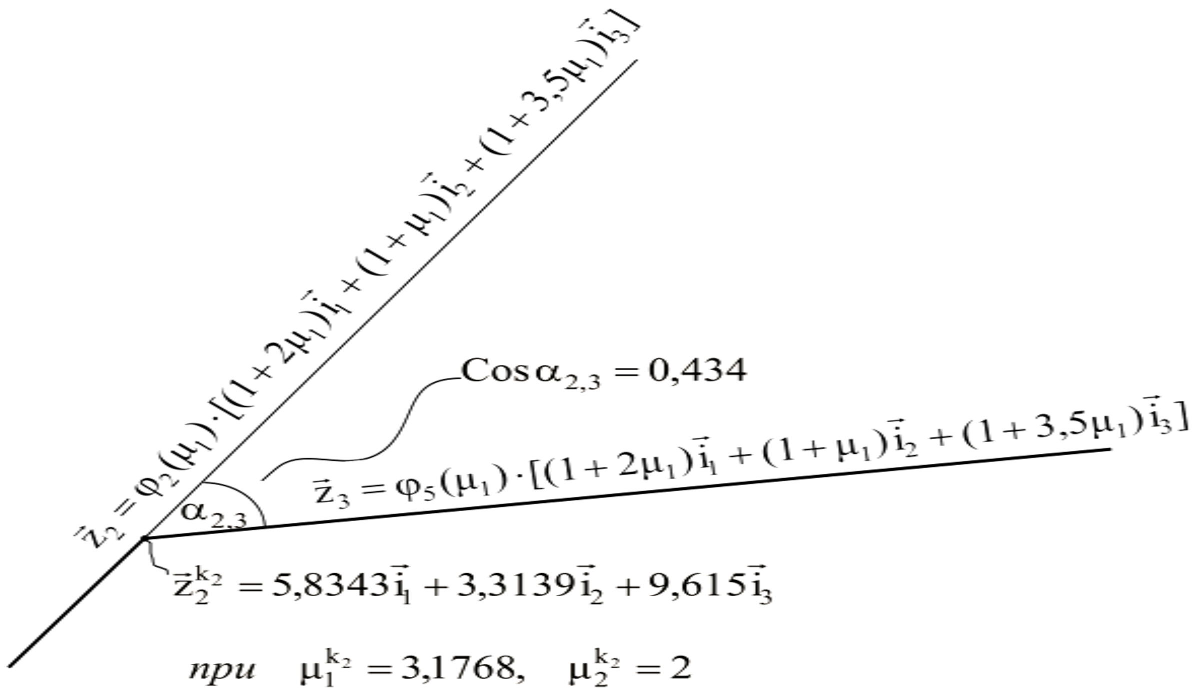

Fig. 5. Numerical construction of three-component piecewise-linear economic-mathematical model in 3-dimensional vector space R_3.

$$\vec{z}_2^{\,k_2} = 5{,}8343\ \vec{i}_1 + 3{,}3139\ \vec{i}_2 + 9{,}615\ \vec{i}_3 \tag{91}$$

Now construct a vector equation for the third piecewise-linear straight line depending on the vector equation of the first piecewise-linear function $\vec{z}_1$, $cos\ \alpha_{1,2}$, $cos\ \alpha_{2,3}$ and also parameter μ_1 corresponding to the parameter μ_3. For that we use the following defining Eqs. (33), (45)–(48):

$$\vec{z}_3 = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \omega_3(\lambda_3, \alpha_{2,3})]\} \quad \text{for} \quad \mu_1 \ge \mu_1^{k_2},\ \mu_2 \ge \mu_2^{k_2} \tag{92}$$

Here the unaccounted parameters influence function $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ is calculated by means of Eq. (43), the unaccounted parameter influence function $\omega_3(\lambda_3, \alpha_{2,3})$ is calculated from Eqs. (46)–(48) in the form:

$$\omega_3(\lambda_3, \alpha_{2,3}) = \lambda_3 \cdot cos\ \alpha_{2,3} \tag{93}$$

$$\lambda_3 = \frac{\mu_3}{\mu_1^{k_1} - \mu_1} \frac{\left|\vec{z}_2 - \vec{z}_2^{\,k_2}\right|\left|\vec{a}_4 - \vec{z}_2^{\,k_2}\right|}{\vec{z}_1(\vec{z}_2 - \vec{z}_2^{\,k_2})} \frac{\vec{z}_1(\vec{z}_1^{\,k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|} \tag{94}$$

$$cos\ \alpha_{2,3} = \frac{(\vec{z}_2(\mu_2) - \vec{z}_2^{\,k_2})(\vec{a}_4 - \vec{z}_2^{\,k_2})}{\left|\vec{z}_2(\mu_2) - \vec{z}_2^{\,k_2}\right|\left|\vec{a}_4 - \vec{z}_2^{\,k_2}\right|},\ \text{for}\ \mu_2 \ge \mu_2^{k_2} \tag{95}$$

$$\mu_3 = (\mu_2 - \mu_2^{k_2}) \frac{(\vec{a}_3 - \vec{z}_1^{\,k_1})(\vec{a}_4 - \vec{z}_2^{\,k_2})}{(\vec{a}_4 - \vec{z}_2^{\,k_2})^2},\ \text{for}\ \mu_2 \ge \mu_2^{k_2} \tag{96}$$

$$A = (\mu_1^{k_1} - \mu_1) \frac{\left|\vec{a}_2 - \vec{a}_1\right|\left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|}{\vec{z}_1(\left|\vec{z}_1^{\,k_1} - \vec{a}_1\right|)} \tag{97}$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \lambda_2^{k_2} \cdot cos\ \alpha_{1,2} \tag{98}$$

Note that calculation of the function $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ is simplified owing to expression Eq. (88), where instead of the parameter μ_1 we should use its value corresponding to the second intersection point, i.e., $\mu_1 = \mu_1^{k_2} = 3{,}1768$. In this case, we get:

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = \omega_2(\lambda_2, \alpha_{1,2})\Big|_{\mu_1 = 3{,}1768} =$$
$$= 0{,}0901\ \cdot \varphi_1(\mu_1)\Big|_{\mu_1 = 3{,}1768} = -0{,}5613 \tag{99}$$

The mathematical Eq. of the relation of the parameter μ_3 with the parameter μ_1 corresponding to the points of the first piecewise-linear straight line, will look like as follows. We have the condition of relation of the parameter μ_2 and the parameter μ_1 in the form Eq. (76). Therefore, substituting Eqs. (76) in Eq. (96), we get:

$$\mu_3 = [1{,}1927\ (\mu_1 - 1{,}5) - \mu_2^{k_2}] \frac{(\vec{a}_3 - \vec{z}_1^{\,k_1})(\vec{a}_4 - \vec{z}_2^{\,k_2})}{(\vec{a}_4 - \vec{z}_2^{\,k_2})^2} \tag{100}$$

Taking into account Eqs. (73), (75), (78), (91) for $\mu_2^{k_2} = 2$, Eq. (100) accepts the form:

$$\mu_3 = 0{,}4216\ \cdot \mu_1 - 1{,}3394 \quad \text{for}\ \mu_3 \ge 0,\ \mu_1 \ge \mu_1^{k_2} = 3{,}1768 \tag{101}$$

Thus, Eq. (101) establishes the numerical relation between the parameters μ_3 and μ_1.

For μ_3 =0, $\mu_1\big|_{\mu_3=0} = \mu_1^{k_2} = 3{,}1768$, i.e., coincides with the value of the parameter $\mu_1^{k_2}$. For any values of the parameter $\mu_3 \geq 0$, the appropriate value of the parameter μ_1 will be greater than 3,1768, i.e., $\mu_1 \geq 3{,}1768$. Substituting Eqs. (73), (75), (90), (91), (101) in Eq. (94), we get the numerical dependence of the unaccounted factors parameter on the third vector straight line λ_3 depending on the parameter μ_1 for $\mu_1 \geq \mu_1^{k_2} = 3{,}1768$ in the form:

$$\lambda_3 = -0{,}2356 \cdot \varphi_3(\mu_1), \text{ for } \mu_1 \geq \mu_1^{k_2} = 3{,}1768 \tag{102}$$

where the expression $\varphi_3(\mu_1)$ is of the form:

$$\varphi_3(\mu_1) = \frac{0{,}4216 \cdot \mu_1 - 1{,}3394}{\varphi_0(\mu_1)} \cdot \frac{\sqrt{\varphi_2^2(\mu_1)\cdot[17{,}25 \mu_1^2 + 13\mu_1 + 3] - 2\varphi_2(\mu_1)[48{,}635 \mu_1 + 18{,}785] + 137{,}4692}}{\varphi_2(\mu_1)\cdot(17{,}25 \cdot \mu_1^2 + 13\mu_1 + 3) - 48{,}635 \cdot \mu_1 - 18{,}763} \tag{103}$$

Substituting Eqs. (73), (90), (91) in Eq. (95), we get the numerical dependence of $cos\ \alpha_{2,3}$ generated between the second and third piecewise-linear straight lines depending on the parameter μ_1 for the values $\mu_1 \geq \mu_1^{k_2} = 3{,}1768$ in the form: $cos\ \alpha_{2,3} = 0{,}1640 \cdot \varphi_4(\mu_1)$, for $\mu_1 \geq 3{,}1768$ (104)

Where the expression $\varphi_4(\mu_1)$ is of the form:

$$\varphi_4(\mu_1) = \frac{2{,}1657[\varphi_2(\mu_1)(1+2\mu_1) - 5{,}8343] + 5{,}6861[\varphi_2(\mu_1)(1+\mu_1) - 3{,}3139] + 0{,}385[\varphi_2(1+3{,}5\mu_1) - 9{,}615]}{\sqrt{[\varphi_2(\mu_1)(1+2\mu_1) - 5{,}8343]^2 + [\varphi_2(\mu_1)(1+\mu_1) - 3{,}3139]^2 + [+\varphi_2(1+3{,}5\mu_1) - 9{,}615]^2}} \tag{105}$$

As the angle between the two straight lines is a constant quantity, we calculate the numerical value of $cos\ \alpha_{2,3}$ for $\mu_1 = 5$. In this case, we have (Fig. 5):

$$cos\ \alpha_{2,3} = 0{,}434 \tag{106}$$

Substituting Eqs. (102) and (106) in Eq. (93), the numerical value of the parameter $\omega_3(\lambda_3, \alpha_{2,3})$ will equal:

$$\omega_3(\lambda_3, \alpha_{2,3}) = -0{,}1022 \cdot \varphi_3(\mu_1) \quad \text{for } \mu_1 \geq 3{,}1768 \tag{107}$$

Now calculate the coefficient A Eq. (97) for $\mu_1 \geq \mu_1^{k_2} = 3{,}1768$. For that we substitute Eqs. (99), (107), and (108) in Eq. (97) and get the following numerical expression of the coefficient A:

$$A = -25{,}875 \frac{\mu_1 - 1{,}5}{9{,}75 + 25{,}875 \cdot \mu_1} = -25{,}875 \cdot \varphi_0(\mu_1), \quad \text{for } \mu_1 \geq \mu_1^{k_2} = 3{,}1768 \tag{108}$$

Or

$$A = -25,875 \cdot \varphi_0(\mu_1) \quad (109)$$

Substituting the numerical values Eqs. (99), (107) and (108) in Eq. (92), we get a vector equation for the third piecewise-linear straight line, expressed by the vector equation of the first piecewise-linear straight-line and the parameter μ_1 in the form (Fig. 5):

$$\vec{z}_3 = \vec{z}_1 \cdot \varphi_5(\mu_1), \text{ for } \quad \mu_1 \geq \mu_1^{k_2} = 3{,}1768 \quad (110)$$

where

$$\varphi_5(\mu_1) = 1 - 11{,}3514 \cdot \varphi_0(\mu_1) \cdot [1 - 0{,}233 \cdot \varphi_3(\mu_1)] \quad (111)$$

or in the form:

$$\vec{z}_3 = \varphi_5(\mu_1) \cdot [(1 + 2\mu_1)\vec{i}_1 + (1 + \mu_1)\vec{i}_2 + (1 + 3{,}5\,\mu_1)\vec{i}_3] \quad (112a)$$

Now investigate the prediction of economic process and its control on the subsequent $V_4(x_1, x_2, x_3)$ small volume of 3-dimensional vector space with regard to unaccounted parameter factors $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3^{k_3}, \alpha_{2,3})$ that hold on the preceding stages of the process [5-16].

And the numerical values of these unaccounted parameters functions $\omega_2(\lambda_2, \alpha_{1,2})$, $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$ are assumed to be known and are given by Eqs. (99), (107), and (88), having the following numerical expressions:

$$\omega_2(\lambda_2, \alpha_{1,2}) = 0{,}0901 \cdot \varphi_1(\mu_1) \quad (113)$$

$$\omega_2(\lambda_2^{k_2}, \alpha_{1,2}) = -0{,}5613 \quad (114)$$

$$\omega_3(\lambda_3, \alpha_{2,3}) = -0{,}1022 \cdot \varphi_3(\mu_1) \quad \text{for } \mu_1 \geq 3{,}1768 \quad (115)$$

where the expressions $\varphi_1(\mu_1)$ and $\varphi_3(\mu_1)$ are represented by Eqs. (86.b) and (103).

Above for the three-component piecewise-linear economic process we have constructed the third piecewise-linear straight line Eq. (110) depending on an arbitrary parameter μ_1 and unaccounted parameters influence spatial functions $\omega_2(\lambda_2, \alpha_{1,2})$ and $\omega_3(\lambda_3, \alpha_{2,3})$. On the other hand, by Eq. (61) we suggested for the three-component case the economic process predicting vector function $\vec{Z}_4(\beta)$ with regard to the introduced unaccounted parameters predicting influence function $\Omega_4(\lambda_4, \alpha_{3,4})$ [5-16]:

$$\vec{Z}_4(\beta) = \vec{z}_1\{1 + A[1 + \omega_2(\lambda_2^{k_2}, \alpha_{1,2}) + \omega_3(\lambda_3^{k_3}, \alpha_{2,3}) + $$
$$+ \Omega_4(\lambda_4, \alpha_{3,4})]\}$$
$$\text{for } \beta = 1,2 \quad (116)$$

$$\Omega_4(\lambda_4, \alpha_{3,4}) = \lambda_4 \cdot \cos \alpha_{3,4} \quad (117)$$

$$\lambda_4 = \frac{\mu_4}{\mu_1^{k_1} - \mu_1} \frac{\left|\vec{z}_3 - \vec{z}_3^{k_3}\right| \left|\vec{a}_5(\beta) - \vec{z}_3^{k_3}\right|}{\vec{z}_1(\vec{z}_3 - \vec{z}_3^{k_3})} \frac{\vec{z}_1(\vec{z}_1^{k_1} - \vec{a}_1)}{\left|\vec{a}_2 - \vec{a}_1\right| \left|\vec{z}_1^{k_1} - \vec{a}_1\right|}, \quad \text{for } \beta = 1,2, \ \mu_3 \geq \mu_3^{k_3}, \mu_4 \geq 0 \qquad (118)$$

$$\mu_4 = (\mu_1 - \mu_3^{k_3}) \frac{(\vec{a}_4 - \vec{z}_2^{k_2})(\vec{a}_5(\beta) - \vec{z}_3^{k_3})}{(\vec{a}_5(\beta) - \vec{z}_3^{k_3})^2}, \quad \text{for } \beta = 1,2, \ \mu_3 \geq \mu_3^{k_3}, \mu_4 \geq 0 \qquad (119)$$

Here, $\omega_2(\lambda_2^{k_2}, \alpha_{1,2})$ has numerical expression Eq. (114), the function $\omega_3(\lambda_3^{k_3}, \alpha_{2,3})$ for the final point of the third piecewise-linear straight line for $\mu_3 = \mu_3^{k_3}$ and its appropriate values $\mu_1 = \mu_1^{k_3}$ is calculated by means of Eq. (115). As the intersection points of the straight lines are given, accept the value of the intersection point k_3 between the third and fourth predicting straight lines in the form $\mu_3^{k_3} = 3$. And define the appropriate value of the parameter $\mu_1^{k_3}$ from the Eq. connecting the parameters μ_1 and μ_3 in the form Eq. (101):

$$\mu_3 = 0,4216 \cdot \mu_1 - 1,3394, \text{ for } \mu_3 \geq 0, \ \mu_1 \geq \mu_1^{k_2} = 3,1768 \qquad (120)$$

Hence

$$\mu_1 = 2,3719 \cdot \mu_3 + 3,1768 \qquad (121)$$

Substituting the value $\mu_3 = \mu_3^{k_3} = 3$ in Eq. (121), define the numerical value of the parameter $\mu_1^{k_3}$ corresponding to the value of the parameter $\mu_3^{k_3} = 3$ at the intersection point of the third piecewise-linear straight line with the predicting fourth straight line in the form: $\mu_1^{k_3} = 10,2926$
(122)

Substituting the numerical value of $\mu_1^{k_3}$ Eq. (122) in Eq. (115), define the numerical value of the unaccounted parameters function $\omega_3(\lambda_3^{k_3}, \alpha_{2,3})$ at the intersection point between the third piecewise-linear straight line and predicting fourth vector straight line. For that as preliminarily, by Eqs. (86.a, b), (90) calculate the functions $\varphi_0(\mu_1)$, $\varphi_1(\mu_1)$ and $\varphi_2(\mu_2)$ for $\mu_1 = \mu_1^{k_3} = 10,2926$, and get:

$$\varphi_0(\mu_1)\Big|_{\mu_1 = 10,2926} = 0,03185 \qquad (123)$$

$$\varphi_1(\mu_1)\Big|_{\mu_1 = 10,2926} = -6,23 \qquad (124)$$

$$\varphi_2(\mu_1)\Big|_{\mu_1 = 10,2926} = 0,639 \qquad (125)$$

Now, substituting the numerical values Eqs. (123)–(125) in Eq. (168), take into account $\mu_1 = 10,2926$ and define the numerical value of the unaccounted parameter function $\omega_3(\lambda_3^{k_3}, \alpha_{2,3})$ at the third intersection point k_3 in the form: $\omega_3(\lambda_3^{k_3}, \alpha_{2,3}) = -0,2172$ (126)

Substituting Eq. (123)–(125) in Eq. (103) allowing for $\mu_1 = 10,2926$ we define the function $\varphi_3(\mu_1)\Big|_{\mu_1 = 10,2926}$ in the form: $\varphi_3(\mu_1)\Big|_{\mu_1 = 10,2926} = 2,125$ (127)

Substituting Eqs. (123) and (127) in Eq. (111), where we accept $\mu_1 = 10,2926$, find the numerical value of $\varphi_5(\mu_1)\Big|_{\mu_1 = 10,2926}$ in the form:

$$\varphi_5(\mu_1)\Big|_{\mu_1 = 10,2926} = 0,8175 \qquad (128)$$

Substituting Eqs. (128) and (74) in Eq. (110) or Eq. (112), where it is accepted $\mu_1 = 10,2926$, find the coordinate expression of the vector point $\vec{z}_3^{k_3}$ in the form (Fig. 6):

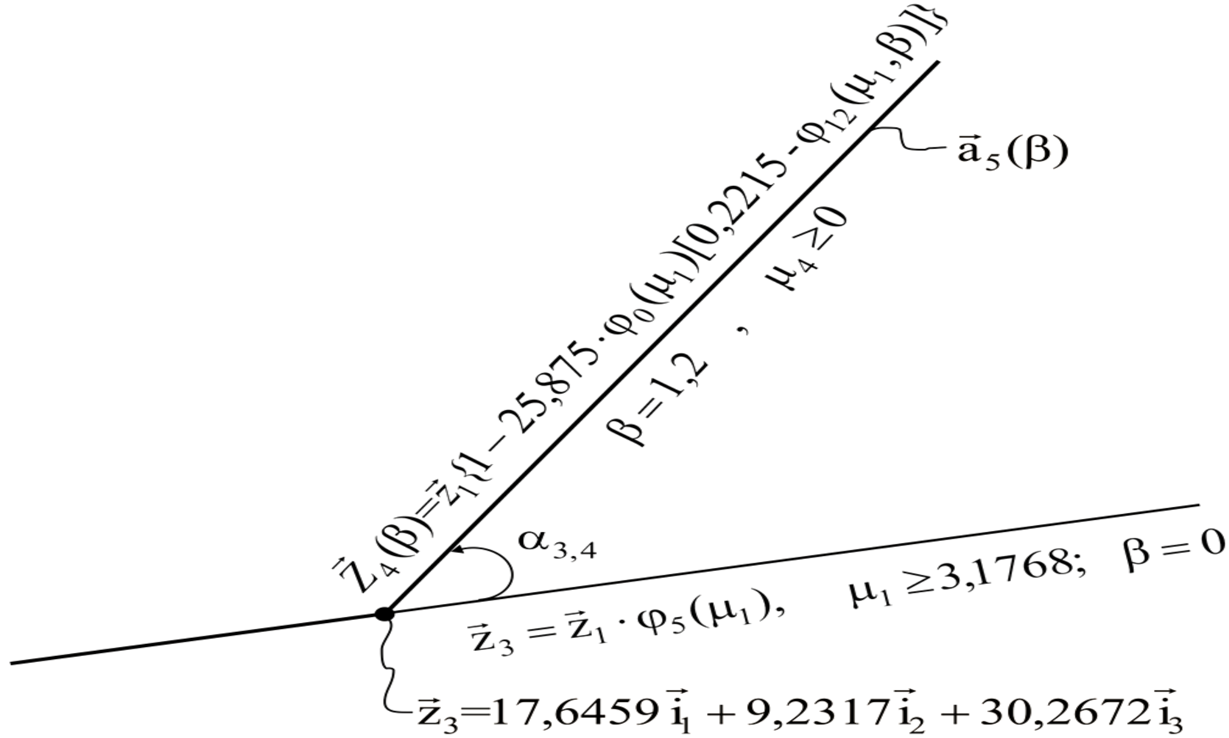

Fig. 6. Numerical construction of the predicting vector function $\vec{Z}_4(\beta)$ with regard to unaccounted parameter influence factor $\Omega_4(\lambda_4, \alpha_{3,4})$ on the base of three-component piecewise-linear economic-mathematical model in3-dimensional vector space R_3.

$$\vec{z}_3^{k_3} = 17,6459 \cdot \vec{i}_1 + 9,2317 \cdot \vec{i}_2 + 30,2672 \cdot \vec{i}_3 \qquad (129)$$

Now, by Eq. (117) calculate the unaccounted parameters predicting function $\Omega_4(\lambda_4, \alpha_{3,4})$. For defining it, as preliminarily we find the numerical dependence of the parameter μ_4 on the parameter μ_1, λ_4, $cos\ \alpha_{3,4}$, and also on the vector $\vec{a}_5(\beta)$ for $\beta = 1,2$. Therewith we note that the vector $\vec{a}_5(\beta)$ for the values $\beta = 1,2$ has coordinate form Eq. (69). Here the coordinate $\vec{a}_{5i}(\beta)$ in 3-dimensional space are determined by Eq. (170). Substituting Eqs. (70), (73), (91), (120) in Eq. (119) for $\mu_3^{k_3} = 3$ establish the numerical dependence of the parameter μ_4 on the parameter μ_1 in the form:

$$\mu_4 = (0,4216 \cdot \mu_1 - 4,3393) \cdot \varphi_6(\mu_1, \beta) \quad \text{for} \quad \mu_1 \geq 10,2926, \ \beta = 1,2 \qquad (130)$$

where

$$\varphi_6(\mu_1, \beta) = \frac{2,1657[a_{51}(\beta) - \varphi_5(1+2\mu_1)] + 5,6861[a_{52}(\beta) - \varphi_5(1+\mu_1)] + 0,385\ [a_{53}(\beta) - \varphi_5(1+3,5\ \mu_1)]}{[a_{51}(\beta) - \varphi_5(1+2\mu_1)]^2 + [a_{52}(\beta) - \varphi_5(1+\mu_1)]^2 + [a_{53}(\beta) - \varphi_5(1+3,5\ \mu_1)]^2} \qquad (131)$$

Substitute Eqs. (168), (73)–(75), (110), (129), and (130) allowing for Eq. (70) in Eq. (118) and define the predicting parameter λ_4 in the form:

$$\lambda_4 = -\varphi_8(\mu_1)\cdot\varphi_6(\mu_1,\beta)\cdot\varphi_7(\mu_1,\beta) \tag{132}$$

where

$$\varphi_7(\mu_1,\beta) = \sqrt{\begin{matrix}[a_{51}(\beta) - 17{,}6459]^2 + [a_{51}(\beta) - 9{,}2317]^2 + \\ + [a_{53}(\beta) - 30{,}2672]^2\end{matrix}} \tag{133}$$

$$\varphi_8(\mu_1) = \frac{\sqrt{\begin{matrix}[(1+2\mu_1)\varphi_5 - 17{,}6459]^2 + \\ + [(1+\mu_1)\varphi_5 - 9{,}2317]^2 + \\ + [(1+3{,}5\mu_1)\varphi_5 - 30{,}2672]^2\end{matrix}}}{\begin{matrix}\varphi_5[(1+2\mu_1)^2 + (1+\mu_1)^2 + (1+3{,}5\mu_1)^2] - \\ - [17{,}6459(1+2\mu_1) + \\ + 9{,}2317(1+\mu_1) + 30{,}2672(1+3{,}5\mu_1)]\end{matrix}} \tag{134}$$

Now define the numerical value of $cos\ \alpha_{3,4}$ generated between the third piecewise-linear function $\vec{z}_3$ and predicting fourth vector function $\vec{Z}_4(\mu_4)$ (Fig. 6) [5-16]:

$$cos\ \alpha_{3,4} = \frac{(\vec{z}_3 - \vec{z}_3^{\,k_3})(\vec{a}_5(\beta) - \vec{z}_3^{\,k_3})}{\left|\vec{z}_3 - \vec{z}_3^{\,k_3}\right|\left|\vec{a}_5(\beta) - \vec{z}_3^{\,k_3}\right|} \tag{135}$$

For that substitute Eqs. (69), (74), (110), and (129) in Eq. (135) and get:

$$cos\ \alpha_{3,4} = \frac{\varphi_9(\mu_1,\beta)}{\varphi_{10}(\mu_1,\beta)\cdot\varphi_{11}(\mu_1,\beta)} \tag{136}$$

where

$$\begin{aligned}\varphi_9(\mu_1,\beta) = {} & a_{51}(\beta)[(1+2\mu_1)\varphi_5 - 17{,}6459] + \\ & + a_{52}(\beta)[(1+\mu_1)\varphi_5 - 9{,}2317] + \\ & + a_{53}(\beta)[(1+3{,}5\mu_1)\varphi_5 - 30{,}2672] - \\ & - \varphi_5[17{,}6459(1+2\mu_1) + 9{,}2317(1+\mu_1) + \\ & + 30{,}2672(1+3{,}5\mu_1)] + 1312{,}7085\end{aligned} \tag{137}$$

$$\varphi_{10}(\mu_1,\beta) = \\ = \sqrt{\begin{matrix}\varphi_5^2[(1+2\mu_1)^2 + (1+\mu_1)^2 + (1+3{,}5\mu_1)^2] - \\ - 2\varphi_5[17{,}6459(1+2\mu_1) + \\ + 9{,}2317(1+\mu_1) + 30{,}2672(1+3{,}5\mu_1)] + 1312{,}7055\end{matrix}} \tag{138}$$

$$\varphi_{11}(\mu_1,\beta) = \sqrt{\begin{matrix}[a_{51}(\beta) - 17{,}6459]^2 + [a_{51}(\beta) - 9{,}2317]^2 + \\ + [a_{53}(\beta) - 30{,}2672]^2\end{matrix}} \tag{139}$$

Substituting Eqs. (132) and (136) in Eq. (117), establish the numerical representation of the unaccounted parameter predicting influence function $\Omega_4(\lambda_4, \alpha_{3,4})$ in the form:

$$\Omega_4(\lambda_4, \alpha_{3,4}) = -\varphi_{12}(\mu_1, \beta) \tag{140}$$

where

$$\varphi_{12}(\mu_1, \beta) = $$
$$= \varphi_6(\mu_1, \beta) \cdot \varphi_7(\mu_1, \beta) \cdot \varphi_8(\mu_1) \cdot \frac{\varphi_9(\mu_1, \beta)}{\varphi_{10}(\mu_1, \beta) \cdot \varphi_{11}(\mu_1, \beta)} \tag{141}$$

Representing the numerical values Eqs. (99), (108), (126), and (140) in Eq. (116), define the concrete form of the predicting vector function on the fourth small volume of 3-dimensional space for $\beta = 1,2$ in the form (Fig. 6):

$$\vec{Z}_4(\beta) = \vec{z}_1\{1 - 25{,}875 \cdot \varphi_0(\mu_1)[1 - 0{,}5613 - 0{,}2172 - \varphi_{12}(\mu_1, \beta)]\}$$

for $\beta = 1,2$

or

$$\vec{Z}_4(\beta) = \vec{z}_1\{1 - 25{,}875 \cdot \varphi_0(\mu_1)[0{,}2215 - \varphi_{12}(\mu_1, \beta)]\}$$

for $\beta = 1,2$ (142)

or in the coordinate form:

$$\vec{Z}_4(\beta) = \{1 - 25{,}875 \cdot \varphi_0(\mu_1)[0{,}2215 -$$
$$- \varphi_{12}(\mu_1, \beta)]\} \cdot [(1 + 2\mu_1)\vec{i}_1 + (1 + \mu_1)\vec{i}_2 + (1 + 3{,}5\,\mu_1)\vec{i}_3]$$

for $\mu_1 \geq 10{,}2926$, $\beta = 1,2$ (143)

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