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Duplication and Switching of Divisor Cordial Graphs

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Abstract

A divisor cordial labeling of a graph G with vertex set V is a bijection f from V to {1, 2, 3, . . .,|V|} such that if an edge uv is assigned the label 1 if f(u) divides f(v) or f(v) divides f(u) and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph has a divisor cordial labeling, then it is called ***divisor cordial graph***. In this paper, we prove that fan graph, switching of a pendant vertex of a helm graph, switching of a vertex of flower graph, switching of closed helm graph, and also duplication of an arbitrary vertex by an edge of a fan graph are divisor cordial.

Keywords: Cordial labeling, divisor cordial labeling, duplication and switching of graphs

1. Introduction

By a graph, we mean a finite undirected graph without loops and multiple edges. For terms not defined here, we refer to Harary [7]. Cordial graph was first introduced by I. Cahit [1] in 1987 as a weaker version of graceful and harmonious graphs and was based on {0, 1} binary labeling of vertices. He showed that (i) every tree is cordial (ii) K_n is cordial if and only if n ≤ 3 (iii) $K_{r,s}$ is cordial for all r and s (iv) the wheel W_n is cordial if and only if n $\equiv 3(\text{mod}4)$ (v) C_n is cordial if and only if n $\not\equiv$ 2(mod4) (vi) an Eulerian graph is not cordial if its size is congruent to 2 modulo 4. Other types of cordial graphs are considered in [3, 7, 10, 11]. For more related results on cordial graphs, one can refer to Gallian [6].

Definition 1.1 [1]

A binary vertex labeling of graph G(V, E), where each edge uv is labeled with |f(u) – f(v)| (mod 2), is called a ***cordial labeling*** if$|v_f(0)–v_f(1)| \leq 1$ and $|e_f(0) – e_f(1)| \leq 1$, where $v_f(i)$denote the number of vertices labeled with i under f and $e_f(i)$ denote the number of edges labeled with i, where i = 0, 1. A graph G is called cordial if it admits a cordial labeling.

Definition 1.2 [15]

Let G be a graph with vertex set V(G) and edge set E(G) and let $f: E(G) \to \{0,1\}$. Define f^*on V(G) by$f^*(v) = \sum \{ f(uv), uv \in E(G)\}(mod2)$. The function f is called an ***E-cordial labeling*** of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called ***E-cordial*** if it admits E-cordial labeling.

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In 1997 Yilmaz and Cahit [15] have introduced E-cordial labeling as a weaker version of edge-graceful labeling. More results are seen in [15,4]

Definition: 1.3 [4]

A ***prime cordial labeling*** of a graph G with vertex set V is a bijection f from V to {1, 2, 3, . . . , |V|} such that if each edge uv is assigned the label 1 if gcd (f(u), f(v)) = 1 and 0 if gcd (f(u), f(v)) > 1, then the number of edges having label 0, and the number of edges having label 1,differ by at most 1. Sundaram et. al [9] has introduced the notion of prime cordial labeling and proved some graphs are prime cordial

Definition: 1.4

The ***fan*** $\boldsymbol{F_n}$ is the graph obtained by taking (n - 2) concurrent chords in cycle C_{n+1}. The vertex at which all the chords are concurrent is called the ***apex vertex***. More precisely, P_n + K_1 is called the ***fan*** $\boldsymbol{F_n}$.

Definition: 1.5

A ***helm*** H_n, $n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each rim vertex.

Definition: 1.6

The ***flower*** Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to apex vertex of the helm.

Definition: 1.7

The ***closed helm*** CH_n is the graph obtained from a ***helm*** H_n by joining each pendant vertex to form a cycle.

Definition 1.8[11]

Let G = (V, E) be a simple graph and f: V→ $\{1, 2, 3, . . . |V|\}$ be a bijection. For each edge uv, assign the label 1 if either f(u) / f(v) or f(v) / f(u) and the label 0 otherwise. Then f is called a ***divisor cordial labeling***. A graph with a divisor cordial labeling is called ***divisor cordial graph*.**

R.Varatharajan, S.Navaneethakrishnan and K.Nagarajan [13], introduced the concept of divisor cordial and proved the graphs such as path, cycle, wheel, star and some complete bipartite graphs are divisor cordial graphs and in [14], they proved some special classes of graphs such as full binary tree, dragon, corona, $G * K_{2,n}$and $G * K_{3,n}$ are divisor cordial. We proved in [8] that some special graphs such as switching of a vertex of cycle, wheel, helm, duplication of arbitrary vertex of cycle, duplication of arbitrary edge of cycle, split graph of $K_{1,n}$, $B_{n,n}$, $B^2_{n,n}$ are divisor cordial graphs.

Labeled graph have variety of application in coding theory, particularly for missile guidance codes, design of good radar type codes and convolution codes with optimal auto correlation properties. Labeled graph plays vital role in the study of X-ray crystallography, communication networks and to determine optimal circuit layouts.

In this paper, we prove that fan graph F_n, switching of a pendant vertex of a helm graph H_n, switching of a vertex of flower graph Fl_n, switching of closed helm graph CH_n and also duplication of a arbitrary vertex by an edge of a fan F_n are divisor cordial.

2. Main results

Theorem: 2.1

The fan F_n is a divisor cordial graph.

Proof:

Let v be the apex vertex and $v_1, v_2, v_3, \dots, v_n$ be the other vertices of the path P_n in fan F_n. Thenp = n + 1 and q = 2n –1. We define vertex labeling f: V→ {1,2, . . ., p} as follows.

$f(v) = 1$
$f(v_i) = i + 1\ ;\ 1 \leq i \leq n$

Since 1 divides any integers, the n edges receive label 1 and the consecutive numbers does not divide each other, so the (n –1) edges in path receive label 0.

Now we observe that $e_f(0) = n - 1; e_f(1) = n$.

Hence $|e_f(0) - e_f(1)| = 1$.

Thus F_n is a divisor cordial graph. ■

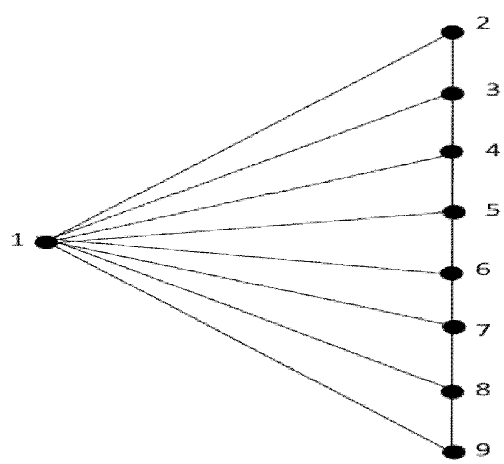

Figure 1: F_8 is a divisor cordial

3. Switching of a vertex

Definition: 3.1[13]

*A **vertex switching G_v*** of a graph G is the graph obtained by taking a vertex v of G, removing the entire edges incident to v and adding edges joining v to every other vertex which is not adjacent to v in G.

Theorem: 3.2

Switching of a pendant vertex in helm graph H_n admits divisor cordial labeling.

Proof:

Let H_n be a helm graph with w as the apex vertex and $u_1, u_2, u_3, \ldots u_n$ be the pendant vertices and $v_1, v_2, v_3, \ldots v_n$ be the vertices of cycle. Let G_{v_n}denote the graph obtained by switching of a pendant vertex u_n of $H_n = G$. Here p = 2n + 1 and q = 5n – 2. Without loss of generality, let us assume that the vertex u_n is switched. We consider two cases.

$$f(u_n) = 1$$
$$f(w) = 2$$
$$f(u_i) = \begin{cases} 2i + 1\ ;\ 1 \le i \le \frac{n}{2} \\ 2(i+1);\ \left(\frac{n}{2} + 1\right) \le i \le n - 1 \end{cases}$$
$$f(v_i) = \begin{cases} 2(i+1); 1 \le i \le \frac{n}{2} \\ 2i + 1;\ \left(\frac{n}{2} + 1\right) \le i \le n - 1 \end{cases}$$

Since the pendant vertex u_n is given label 1, the 2(n - 1) + 1 edges incident to it receives label 1. Also, since the apex vertex is given label 2 and cyclic vertex v_i; $1 \le i \le \frac{n}{2}$ is labeled with even integers, they divide each other and receive label 1. All other edges receive label 0. That is, $2(n-1) + 1 + \frac{n}{2}$ edges receive label 1.

Hence $e_f(1) = e_f(0) = \frac{5n-2}{2}$

Therefore, $|e_f(1) - e_f(0)| = 0$

Case (ii) n is odd

$$f(u_n) = 1$$
$$f(w) = 2$$
$$f(v_i) = \begin{cases} 2(i+1)\ ; 1 \le i \le \lfloor \frac{n}{2} \rfloor \\ 2i + 1\ ;\ \lceil \frac{n}{2} \rceil \le i \le n \end{cases}$$
$$f(u_i) = \begin{cases} 2i + 1\ ; 1 \le i \le \lfloor \frac{n}{2} \rfloor \\ 2(i+1);\ \lceil \frac{n}{2} \rceil \le i \le n \end{cases}$$

As the above case, we observe that $(2n - 1) + \lceil \frac{n}{2} \rceil$ edges receive label1.

That is, $e_f(1) = \frac{5n-3}{2}$ edges receive label 1and $e_f(0) = \frac{5n-1}{2}$ edges receive label 0

Hence, $\left|e_f(0) - e_f(1)\right| = 1$

From both the cases $|e_f(0) - e_f(1)| \le 1$

Hence switching of a pendant vertex of H_n is a divisor cordial graph.■

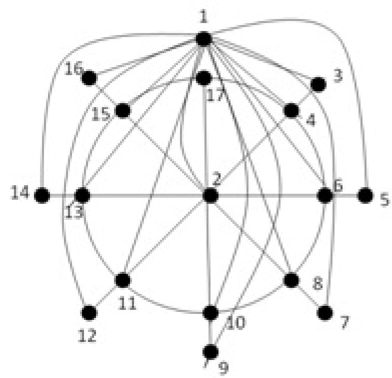

Figure 2: Switching of a pendant vertex in H_8

Note: 3.3

Switching of a vertex of inner cycle of a helmH_n, the graph becomes disconnected.

Theorem: 3.4

The graph obtained by switching of a vertex of flower graph Fl_n is divisor cordial.

Proof:

Let w be the apex vertex; $v_1, v_2, v_3, \dots, v_n$ be the vertices of cycle and $u_1, u_2, u_3, \dots, u_n$ be the pendant vertices. We define f: V(G) → $\{1, 2, \dots$,p}

Case (i) Switching of a vertexu_i, $1 \leq i \leq n$.

Without loss of generality let us assume that the rim vertex u_n is switched. Then p = 2n + 1, q = 3(2n – 1)

$$f(u_n) = 1$$
$$f(w) = 2$$
$$f(v_i) = 2\,(i + 1);\ \ 1 \leq i \leq n - 1$$
$$f(u_i) = 2\,i + 1\,;\ \ 1 \leq i \leq n - 1$$
$$f(v_n) = f(u_{n-1}) + 2$$

Since u_n is given label 1 the edges adjacent to u_n receives label 1 the apex vertex is labeled with 2, and v_i ; $1 \leq i \leq n$ is labeled with even integers, since they divide each other, that edges receive label 1. Hence (2n – 1 + n) edges label 1 and other (3n – 1) edges receive label 0.

That is, $e_f(1) = e_f(0) = (3n - 1)$

Therefore, $|e_f(0) - e_f(1)| = 0$

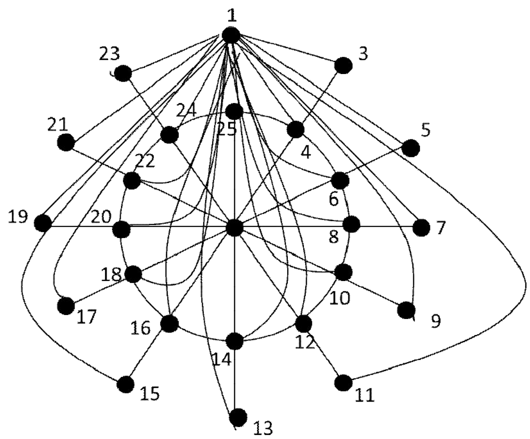

Figure 3: Switching of a pendant vertex of *Fl_{12}*

Case (ii) Switching of a vertex v_i, $1 \leq i \leq n$

Let us assume that the vertex v_1 is switched. Here p = 2n + 1and q = 6n - 8.

$f(w) = 1$

$f(v_i) = 2i\,; 1 \leq i \leq n$
$f(u_i) = 2i + 1\,; 1 \leq i \leq n$

Since the apex vertex is given label 1, the (2n - 1) edges incident to it receives label 1. The switched vertex v_1 is given label2, and so the (n - 3) vertices incident to v also receives label 1. Hence (n - 3) + (2n - 1) = 3n - 4 edges receives label 1 and other (3n - 4) edges receives label 0

That is, $e_f(1) = e_f(0) = (3n - 4)$
Therefore, $|e_f(0) - e_f(1)| = 0$

From both the cases $|e_f(0) - e_f(1)| \leq 1$. Hence graph obtained by switching of a vertex of flower graph Fl_n is divisor cordial.

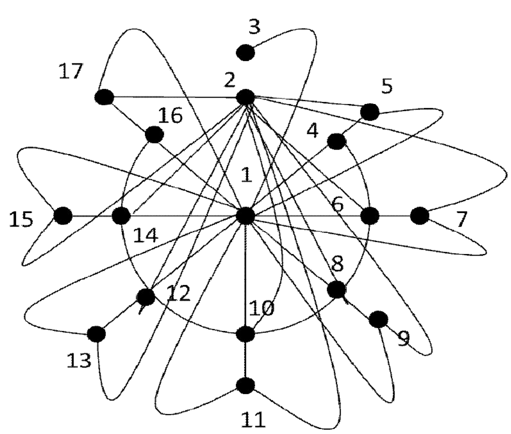

Figure: 4: switching of a vertex v_1 in Fl_8

Case (iii) Switching of the apex vertex w, the graph becomes disconnected.■

Theorem: 3.5

The graph obtained by switching of a vertex of a closed helm graph is integer cordial.

Proof:

Let w be the apex vertex; $v_1, v_2, \ldots v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots u_n$ be the vertices of outer cycle. Define f : V→ {1,2, . . .,p}

Case (i) Switching of a vertex u_i; $1 \leq i \leq n$.

Here p = 2n + 1 and q = 6(n - 1).Without loss of generality let us assume the vertex u_1 is switched

The labeling is as follows:

$f(w) = 2$; $f(v_n) = 2n + 1$

$f(v_i) = 2(i + 1); 1 \leq i \leq n - 1$

$f(u_i) = 2i - 1;\ 1 \leq i \leq n$

Since the vertex u_1 is given label 1, the (2n - 3) edges incident to it receive label 1. Similarly the apex vertex w is given label 2 and the vertices of inner cycle v_i are given even integers and so (n - 2) edges and the edge $(u_2 v_2)$ receives label 1. Other edges receive label 0.

That is, $e_f(1) = e_f(0) = 3(n - 1)$

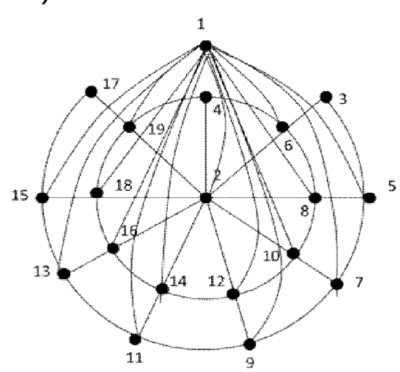

Figure: 5 Switching of u_1 in CH_9

Case (ii) Switching of a vertex v_i; $1 \leq i \leq n$

Without loss of generality let us assume the vertex v_1 is switched. Here p = 2n + 1 and q = 6n – 8

The labeling is as follows:

$f(w) = 2; f(v_1) = 1; f(u_1) = 2n + 1$
$f(v_i) = 2i\ ;\ 2 \leq i \leq n$
$f(u_i) = 2i - 1\ ; 2 \leq i \leq n$. Now interchange u_2 and u_3.

From the above labeling, we observe that (3n - 4) edges receive label 1 and (3n - 4) edges receive label 0.

That is, $e_f(1) = e_f(0) = 3n - 4$.

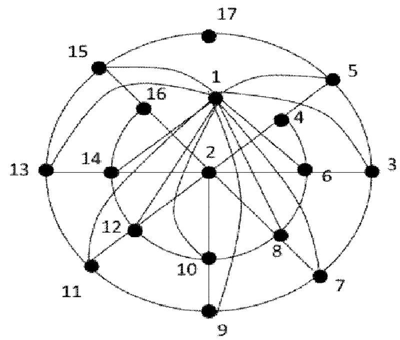

Figure: 6 Switching of vertex v_1.

Case (iii) Switching of apex vertex w

Let the apex vertex w be switched. Then p = 2n + 1 and q = 4n.

Subcase (i) n≢ 2 (mod 4)

The labeling is as follows:

$f(w) = 1$

The vertices of inner cycle $v_1, v_2, \dots v_n$ and rim vertices $u_1, u_2, \dots u_n$ be labeled in the following order

$2, 2 \times 2, 2 \times 2^2, \dots\ 2 \times 2^{k_1}$
$3,\ 3 \times 2, 3 \times 2^2, \dots 3 \times 2^{k_2}$ ….. ….. ….. (2)
$5, 5 \times 2, 5 \times 2^2, \dots 5 \times 2^{k_3}$
. . . .

Where $(2m - 1)\ 2^{k_m} \le n$ and $m \ge 1, k_m > 0$. We observe that $(2m-1)2^a$ divide $(2m - 1)\ 2^b$; $(a < b)$and $(2m - 1)\ 2^{k_i}$ does not divide (2m+1). Now interchange the labels of u_1 and u_2.

The remaining pendant vertices are labeled continuously other than the above labels.

From the labeling, we observe that 2n edges receives label 1 and 2n edges receive label 0.

That is, $e_f(1) = e_f(0) = 2n$.

Subcase (ii) n $\equiv 2 \pmod 4$

The labeling is as follows:

f(w)=1

The vertices of inner cycle $v_1, v_2, \dots v_n$ and pendant vertices $u_1, u_2, \dots u_n$ be labeled in the following order

$2, 2 \times 2, 2 \times 2^2, \dots\ 2 \times 2^{k_1}$
$3, 3 \times 2, 3 \times 2^2, \dots\ 3 \times 2^{k_2}$ ….. ….. ….. (2)
$5, 5 \times 2, 5 \times 2^2, \dots\ 5 \times 2^{k_3}$ . . . .

Where $(2m-1)\ 2^{k_m}\ \leq\ n$ and $m\ \geq\ 1, k_m\ >\ 0$. We observe that $(2m-1)2^a$ divide $(2m-1)\ 2^b; (a\ <\ b)$ and $(2m-1)\ 2^{k_i}$ does not divide (2m+1).

The remaining pendant vertices are labeled continuously other than the above labels.

From the labeling we observe that 2n edges receives label 1 and 2n edges receive label 0.

That is, $e_f(1)\ =\ e_f(0)\ =\ 2n$.

From all the cases $|\ e_f(0)\ -\ e_f(1)\ |\ =\ 0$

Hence the graph obtained by switching a vertex is divisor cordial. ■

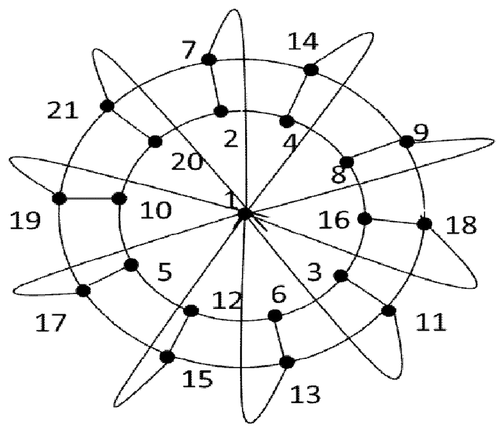

Figure: 6 Switching of apex vertex in Fl_{10}

4. Duplication of a vertex and duplication of an edge.

Definition: 4.1 [**12**]

Duplication of a vertex v_k by a new edge $e\ =\ v'v''$ in a graph G produces a new graph G' such that $N(v')\ =\ \{v_k, v''\}$ and $N(v'')\ =\ \{v_k, v'\}$

Definition: 4.2**[12]**

Duplication of an edge $e\ =\ v_iv_{i+1}$ by a vertex v' in a graph G produces a new graph G' such that $N(v')\ =\ \{v_i, v_{i+1}\}$.

Theorem: 4.3

Duplication of a vertex by an edge in a fan graph F_n is divisor cordial graph.

Proof:

Let v be the apex vertex and $v_1, v_2, v_3, \dots, v_n$ be the other vertices of fan F_n and let v_1' and v_2' be the newly added vertex. Then p = n + 3 and q = 2(n + 1). We define vertex labeling as f: V →{1,2,…, p}as follows.

Case(i)

Let the apex vertex v be duplicated.

Subcase (i) n is even

f(v)=2

Let $v_1, v_2, v_3, \ldots v_n$ be the vertices of path P_n. We label these vertices as follows:

$1, 2 \times 2, 2 \times 2^2, \ldots, 2 \times 2^{k_1}$
$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k_2}$ ….. ….. ….. (2)
$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k_3}$
. . . .

where $(2m-1)\ 2^{k_m} \leq n$ and $m \geq 1$, $k_m > 0$. We observe that $(2m-1)2^a$ divide $(2m-1)\ 2^b$; $(a < b)$and $(2m-1)\ 2^{k_i}$does not divide (2m+1).

The remaining vertices are given other labels up to p. Then (n + 1) edges receives label 1 and (n + 1) edges receives label 2.

That is, $e_f(1) = e_f(0) = (n+1)$

Therefore, $|e_f(0) - e_f(1)| = 0$

Subcase (ii) n is odd

We label as above case. The vertices of path is labeled as

$1, 2 \times 2, 2 \times 2^2, \ldots, 2 \times 2^{k_1}$
$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k_2}$ ….. ….. ….. (2)
$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k_3}$
. . . .
$\lfloor \frac{n}{2} \rfloor, \lfloor \frac{n}{2} \rfloor \times 2, \lfloor \frac{n}{2} \rfloor \times 2^2, \ldots \lfloor \frac{n}{2} \rfloor \times 2^{k_m}$

where $(2m-1)\ 2^{k_m} \leq n$ and $m \geq 1, k_m > 0$. We observe that $(2m-1)2^a$divide $(2m-1)\ 2^b$; $(a < b)$and $(2m-1)\ 2^{k_i}$does not divide (2m+1).

Then$e_f(1) = e_f(0) = (n+1)$
Therefore, $|e_f(0) - e_f(1)| = 0$

Case(ii)

Let any of the vertex $v_i; 1 \leq i \leq n$ be duplicated.

Subcase (i) n is even

$f(v) = 2$; $f(v') = p_1$ and $f(v'') = p_2$; where p_1 and p_2 are prime numbers Let $v_1, v_2, v_3, \ldots v_n$ be the vertices of path P_n. We label these vertices as follows:

$1, 2 \times 2, 2 \times 2^2, \ldots, 2 \times 2^{k_1}$
$3, 3 \times 2, 3 \times 2^2, \ldots, 3 \times 2^{k_2}$ ……………. (2)
$5, 5 \times 2, 5 \times 2^2, \ldots, 5 \times 2^{k_3}$. . . . .

Where $(2m-1)\, 2^{k_m} \leq n$ and $m \geq 1, k_m > 0$. We observe that $(2m-1)2^a$ divide $(2m-1)\, 2^b$; $(a < b)$ and $(2m-1)\, 2^{k_i}$ does not divide $(2m+1)$. Remaining vertices are given other labels other than p_1 and p_2.

Subcase (ii) n is odd

We label as in the above case. The vertices of path are labeled upto $\lfloor \frac{n}{2} \rfloor$, $\lfloor \frac{n}{2} \rfloor \times 2$, $\lfloor \frac{n}{2} \rfloor \times 2^2, \ldots, \lfloor \frac{n}{2} \rfloor \times 2^{k_m}$

From the above cases, we observe that $e_f(1) = e_f(0) = (n+1)$

Therefore, $|e_f(0) - e_f(1)| = 0$

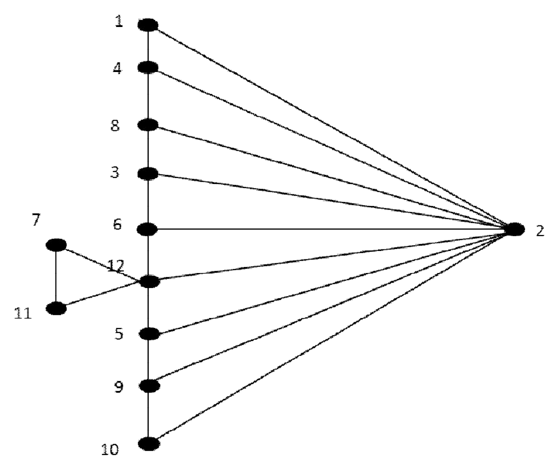

Figure: 6 Duplication of the vertex v_6 in F_9

Case (iii) If the vertex v_1 is duplicated then the labeling as follows;

$f(v) = 2; f(v_1) = p_1; f(v') = 1; f(v'') = p_2$ Where p_1 and p_2 are prime numbers.

Other labels are given as in above cases. From the labeling, we observe that $e_f(1) = e_f(0) = (n+1)$

Therefore, $|e_f(0) - e_f(1)| = 0$

From all cases $|e_f(0) - e_f(1)| \leq 1$.

Hence duplication of a vertex by an edge is a divisor cordial graph.■

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