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On The Distribution Functions of the Range and the Quasi-Range of the Negatively Skewed Extended Generalized Logistic Distribution

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Abstract

In this paper, we obtain the distribution functions of the range and the quasi-range of the random variables from the negatively skewed extended generalized logistic distribution. The covariance between the m^{th} and the q^{th} order statistics $m > q$ is also presented. 2000 Mathematics Subject Classifications: Primary 62E15; Secondary 62G30.

Keyword: Covariance, Negatively Skewed Extended Generalized Logistic Distribution, Order Statistics,Quasi-Range And Range.

1. Introduction

The probability density function of the logistic distribution in its reduced form for a continuous random variable X is

$$f_X(x) = \frac{e^{-x}}{(1+e^{-x})^2}, -\infty < X < \infty, \qquad (1.1)$$

While it's cumulative distribution function is

$$F_{X(x)=}(1+e^{-x})^{-1}, \qquad -\infty<X<\infty \qquad (1.2)$$

The shape of this distribution that is similar to that of normal distribution has made it to be preferred to normal distribution by some researchers like Berkson (1944,1950,1953), Berkson and Hodges (1960) etc. Ojo (1989) used the logistic model to analyze some social problem data. Researchers have been working on order statistics from the logistic distribution over time. It was considered in Plackett (1958), Birnbaum and Dudman (1963), Tarter and Clark (1965), Shuah (1966, 1970), Gupta, Qureishi and Shah (1967) obtain best unbiased estimated estimators of the location and scale parameters of the logistic distribution using order statistics. It is well known that range and quasi-range are important statistics that depend on order statistics. Gupta and Shah (1965) obtained the distribution of the range from the logistic distribution while Malik (1980) obtained the distribution function of the quasi-range from the logistic distribution.

In recent times, researchers have focused more on generalizing probability distribution functions with the aim of making the functions to be more robust and applicable in modeling different types of data. The logistic distribution has enjoyed the practice of generalization in many forms as could be seen in George and Ojo (1980), Balakrishnan and Leung (1988a), Wu et al (2000), Olapade (2004,2005,2006). Though, many works had been done on the study of order statistics from the logistics distribution, many of the various generalizations of the distribution have not enjoyed such privilege. Balakrishnan and Leung (1988a) studied order statistics from the type I generalized logistic distribution while Balakrishnan and Leung (1988b) obtained the means, variances and covariance of order statistics, BLUE's for the type I generalized logistic distribution.

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In this paper, we will obtain the distributions of the range, the quasi-range and the covariance functions from negatively skewed extended generalized logistic distribution with probability density function

$$f_X(x;\lambda,p) = \frac{\lambda p e^{-px}}{(\lambda+e^{-x})^p}, \qquad -\infty < X < \infty, \qquad p > 0, \lambda > 0 \qquad (1.3)$$

And cumulative distribution function

$$F_X(x;\lambda,p) = 1 - \frac{e^{-px}}{(\lambda+e^{-x})^p}, \qquad -\infty < X < \infty, \qquad p > 0, \lambda > 0 \qquad (1.4)$$

Some theorems that characterize this distribution were presented in Olapade (2005) while some properties and applications of this distribution were presented in Olapade (2009) who obtained the distribution of the r^{th} order statistics and established the probability density functions of the maximum and minimum order statistics from this distribution.

2. Distribution of the range from the negatively skewed extended generalized logistic distribution

Given a set of random variables $X_1, X_2, \dots, X_n$ of size n from the negatively skewed extended logistic distribution, let $X_{1:n} \le X_{2:n} \le \cdots X_{n:n}$ be the corresponding order statistics. Let $F_{X_{r:n}(x)}$, $r = 1,2,\dots,n$ be the cumulative distribution function of the r^{th} order statistics $X_{r:n}$ and $f_{X_{r:n}}(x)$ denote its probability density function. David (1970) gives the probability density function of $X_{r:n}$ as

$$f_{X_{r:n}}(x) = \frac{1}{B(r,n-r+1)}[F(x)]^{r-1}[1-F(x)]^{n-r}f(x), \qquad (2.1)$$

Where $B(.,.)$ is a complete better function. For the negatively skewed extended generalized logistic distribution with probability density function and cumulative distribution function given in equation (1.3) and (1.4) respectively. Let us define the sample range W_n by $W_n = X_{n:n} - X_{1:n}$, the cumulative distribution of W_n can be written as

$$P_r(W_n \le w) = n\int_{-\infty}^{\infty}\left(F(x+w) - F(x)\right)^{n-1} f(x)dx \qquad (2.2)$$

Now we substitute for $F(\,.\,)$ and $f(\,.\,)$ in equation (2.2) to obtain

$$P_r(W_n \le w) = n\int_{-\infty}^{\infty}\left(\frac{e^{-px}}{(\lambda+e^{-x})^p} - \frac{e^{-p(x+w)}}{(\lambda+e^{-x-w})^p}\right)^{n-1} \frac{\lambda e^{-px}}{(\lambda+e^{-x})^{p+1}}dx \qquad (2.3)$$

$$= np\lambda\int_{-\infty}^{\infty}\left(\frac{1}{(\lambda+e^{-x})^p} - \frac{e^{-pw}}{(\lambda+ae^{-x})^p}\right)^{n-1} \frac{e^{-npx}}{(\lambda+e^{-x})^{p+1}}dx \qquad (2.4)$$

Where $a = e^{-w}$,then

$$P_r(W_n \le w) = np\lambda\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\int_{-\infty}^{\infty}\left(\frac{1}{(\lambda+e^{-x})^p}\right)^{n-1-k}\left(\frac{e^{-w}}{\lambda+ae^{-x}}\right)^{pk}\frac{e^{-npx}}{(\lambda+e^{-x})^{p+1}}dx \qquad (2.5)$$

$$= np\lambda a^{pk}\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\int_{-\infty}^{\infty}\frac{e^{-npx}}{(\lambda+ae^{-x})^{pk}(\lambda+e^{-x})^{np-pk+1}}dx \qquad (2.6)$$

Let $t = (\lambda+e^{-x})^{-1}$, then

$$P_r(W_n \le w) = np\lambda a\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\int_{0}^{1/\lambda}\frac{(1-\lambda t)^{np-1}}{(1+\lambda(a-1)t)^{np-pk+1}}dx \qquad (2.7)$$

$$= np\lambda a\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\int_0^{1/\lambda}\frac{(1-\lambda t)^{np-1}}{(1+bt)^{np-pk+1}}dt = np\lambda a\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}A(n,p,k,\lambda), \quad (2.8)$$

Where $b=\lambda(a-1)$ and

$$A(n,p,k,\lambda) = \int_0^{1/\lambda}(1-\lambda t)^{np-1}(1+bt)^{-(np-pk+1)}dt$$

Let $v = 1+bt$, then

$$A(n,p,k,\lambda) = 1/b\int_0^{1+b/\lambda}(1-\lambda(v-1)/b)^{np-1}v^{pk-np-1}dv$$

$$= 1/b\sum_{j=0}^{np-1}(-1)^j\binom{np-1}{j}\lambda^j\int_1^{1+b/\lambda}\left(\frac{v-1}{b}\right)^j v^{pk-np-1}dv$$

$$= 1/b\sum_{j=0}^{np-1}(-1)^j\binom{np-1}{j}\lambda^j\left(\frac{1}{-b}\right)^j\int_1^{1+b/\lambda}v^{pk-np-1}(1-v)^j dv$$

$$= \sum_{j=0}^{np-1}(-1)^{j-1}\binom{np-1}{j}\frac{\lambda^j}{(-b)^{j+1}}\int_1^{1+b/\lambda}\sum_{i=0}^{j}(-1)^i\binom{j}{i}v^{i+pk-pn-1}dv \quad (2.9)$$

$$= \sum_{j=0}^{np-1}(-1)^{j-1}\binom{np-1}{j}\frac{\lambda^j}{(-b)^{j+1}}\left[(-1)^{p(n-k)}\binom{j}{p(n-k)}In\left(1+\frac{b}{\lambda}\right)+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^i\binom{j}{i}\frac{\left(1+\frac{b}{\lambda}\right)^{i+pk-pn}-1}{i+pk-pn}\right] \quad (2.10)$$

$$= \sum_{j=0}^{np-1}(-1)^{j-1}\binom{np-1}{j}\frac{\lambda^j}{(-b)^{j+1}}\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}w+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^i\binom{j}{i}\frac{e^{-w(i+pk-pn)}-1}{i+pk-pn}\right] \quad (2.11)$$

Substitute for $A(n,p,k,\lambda)$ in equation (2.8), we have

$$P_r(W_n\leq w) = np\lambda a\sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}\sum_{j=0}^{np-1}(-1)^{j-1}\binom{np-1}{j}\frac{\lambda^j}{(-b)^{j+1}}\times$$

$$\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}w+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^i\binom{j}{i}\frac{e^{-w(i+pk-pn)}-1}{i+pk-pn}\right] \quad (2.12)$$

$$= np\sum_{k=0}^{n-1}\sum_{j=0}^{np-1}(-1)^{k+j-1}\binom{n-1}{k}\binom{np-1}{j}\frac{e^{-w}}{(1-e^{-w})^{j+1}}$$
$$\times\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}w+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^i\binom{j}{i}\frac{e^{-w(i+pk-pn)}-1}{i+pk-pn}\right] \quad (2.13)$$

$$= np\sum_{k=0}^{n-1}\sum_{j=0}^{np-1}(-1)^{k+j-1}\binom{n-1}{k}\binom{np-1}{j}\frac{1}{(1-e^{-w})^{j+1}}$$
$$\times\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}we^{-w}\right.$$
$$\left.+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^{i}\binom{j}{i}\frac{e^{-w(i+pk-pn)}-e^{-w}}{i+pk-pn}\right]. \qquad (2.14)$$

By differentiating the distribution function of the sample range in equation (2.14) with respect to w, we derive the density function of the sample range W_n as

$$P(w) = np\sum_{k=0}^{n-1}\sum_{j=0}^{np-1}(-1)^{k+j}\binom{n-1}{k}\binom{np-1}{j}\frac{(j+1)e^{-w}}{(1-e^{-w})^{j+2}}$$
$$\times\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}we^{-w}+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^{i}\binom{j}{i}\frac{e^{-w(i+pk-pn+1)}-e^{-w}}{i+pk-pn}\right]+$$
$$np\sum_{k=0}^{n-1}\sum_{j=0}^{np-1}(-1)^{k+j+1}\binom{n-1}{k}\binom{np-1}{j}\frac{1}{(1-e^{-w})^{j+1}}\times\left[(-1)^{pn-pk+1}\binom{j}{p(n-k)}(1-w)e^{-w}\right]$$
$$+\sum_{i=0,i\neq p(n-k)}^{j}(-1)^{i+1}\binom{j}{i}\frac{(i+pk-pn+1)e^{-w(i+pk-pn+1)}-e^{-w}}{i+pk-pn}. \qquad (2.15)$$

It could be noted that the $F_w(w)$ and $f_w(w)$ of the negatively skewed extended generalized logistic distribution are independent of λ but they depend on p.

3 Distribution of the quasi-range of random variables from the negatively skewed extended generalized logistic distribution

Given a set of random variables $X_1, X_2, \dots, X_n$ of size n negatively skewed extended generalized logistic distribution, let $X_{1:n} \leq X_{2:n} \leq \cdots X_{r:n} \leq X_{r+1:n} \leq \cdots X_{n:n}$ be the corresponding order statistics. Let us define the sample r^{th}quasi-range W by

$$W = X_n - X_{r+1:n}, \quad r = 0,1,\dots,(n-1)/2, \qquad (3.1)$$

where n is odd. The joint probability density functions of $X_{r+1:n}$ and $X_{n-r:n}$ is given by

$$f(x_{r+1:n}, x_{n-r:n}) = \frac{n!}{r!\,(n-2r-2)!\,r!}[F(x_{r+1:n})]^{r}[F(x_{n-r:n})-F(x_{r+1:n})]^{n-2r-2}\times$$
$$[1-F(x_{n-r:n})]^{r}f(x_{n-r:n})f(x_{r+1:n}), -\infty < X_{r+1:n} < X_{n-r:n} < \infty. \qquad (3.2)$$

Since $X_{n-r:n}= X_{r+1:n} + W$, we have

$$P_r(W_n \leq w) = \int_{-\infty}^{\infty}\int_{x_{r+1:n}}^{x_{r+1:n}+w} f(x_{r+1:n}, x_{r+1:n}+w)d(x_{r+1:n}+w)dx_{r+1:n}. \qquad (3.3)$$

Let $x_{r+1:n} = x$ (for convenience),

$$P_r(W \leq w) = \frac{n!}{r!\,(n-2r-2)!\,r!}\int_{-\infty}^{\infty}[F(x)]^{r}f(x)dx\ \times$$

$$\int_{x}^{x+w}[1-F(x+w)]^{r}[F(x+w)-F(x)]^{n-2r-2}f(x+w)d(x+w). \qquad (3.4)$$

Let $y = F(x+w)$ and $dy = F'(x+w)d(x+w)$, then

$$P_r(W \le w) = \frac{n!}{r!\,(n-2r-2)!\,r!}\int_{-\infty}^{\infty}[F(x)]^{r}f(x)dx \times$$
$$\{\int_{F(x)}^{F(x+w)}[1-y]^{r}[y-F(x)]^{n-2r-2}dy\} \qquad (3.5)$$

Integrating the expressions in braces r times by parts, we have

$$P_r(W \le w) = \sum_{k=0}^{r}\prod_{i=0}^{2r-k}\frac{(n-i)}{r!\,(r-k)!}\int_{-\infty}^{\infty}[F(x)]^{r}\,[1-F(x+w)]^{r-k}[F(x+w)-F(x)]^{n-2r+k-1}f(x)dx$$
$$= \sum_{k=0}^{r}\prod_{i=0}^{2r-k}\frac{(n-i)}{r!\,(r-k)!}\sum_{j=0}^{n-2r+k-1}(-1)^{j}\binom{n-2r+k-1}{j}\sum_{l=0}^{r-k}(-1)^{l}\binom{r-k}{l}$$
$$\times\int_{-\infty}^{\infty}[F(x)]^{r+j}\,[F(x+w)]^{n-2r+k-j+l-1}f(x)dx \qquad (3.6)$$

Let $\Lambda = \int_{-\infty}^{\infty}[F(x)]^{r+j}\,[F(x+w)]^{n-2r+k-j+l-1}f(x)dx$, using the $f(x;\lambda,p)$ and $F(x;\lambda,p)$ shown in equations (1.3) and (1.4) respectively, we have

$$\Lambda = \int_{-\infty}^{\infty}\left[1-\frac{e^{-px}}{(\lambda+e^{-x})^{p}}\right]^{(r+j)}\left[1-\frac{e^{-p(x+w)}}{(\lambda+e^{-x-w})^{p}}\right]^{n-2r+k-j+l-1}\frac{\lambda p e^{-px}}{(\lambda+e^{-x})^{p+1}}dx$$
$$= \int_{-\infty}^{\infty}\sum_{h=0}^{r+j}(-1)^{h}\binom{r+j}{h}\left(\frac{e^{-x}}{\lambda+e^{-x}}\right)^{ph}\sum_{m=0}^{n-2r+k-j+l-1}(-1)^{m}\binom{n-2r+k-j+l-1}{m}$$
$$\times\left(\frac{e^{-x-w}}{\lambda+e^{-x-w}}\right)^{pm}\left(\frac{\lambda p e^{-px}}{(\lambda+e^{-x})^{p+1}}\right)dx \qquad (3.7)$$
$$= \sum_{h=0}^{r+j}\sum_{m=0}^{n-2r+k-j+l-1}(-1)^{h+m}\binom{r+j}{h}\binom{n-2r+k-j+l-1}{m}\lambda p e^{-pmw}$$
$$\times\int_{-\infty}^{\infty}\frac{e^{-xp(h+m+1)}}{(\lambda+e^{-x})^{ph+p+1}(\lambda+ae^{-x})^{pm}}dx, \qquad (3.8)$$

where $a = e^{-w}$. Let the integral be Λ_1 and let $t = (\lambda+ae^{-x})^{-1}$, then

$$\Lambda_1 = a^{1-pm}\int_{0}^{1/\lambda}\frac{(1-\lambda t)^{ph+pm+p-1}}{(1+\lambda t(a-1))^{ph+p+1}}dt.$$

Let $y = \lambda t$,

$$\Lambda_1 = \frac{a^{1-pm}}{\lambda}\int_{0}^{1}\frac{(1-y)^{ph+pm+p-1}}{(1+y(a-1))^{ph+p+1}}dy$$
$$= \frac{a^{1-pm}}{\lambda}\int_{0}^{1}(1-y)^{ph+pm+p-1}\sum_{i=0}^{\infty}(-1)^{i}\binom{ph+p-i}{i}(a-1)^{i}y^{i}dy$$

$$= \frac{a^{1-pm}}{\lambda}\sum_{i=0}^{\infty}(-1)^i\binom{ph+p-i}{i}(a-1)^i\int_0^1(1-y)^{ph+pm+p-1}y^i dy$$
$$= \frac{a^{1-pm}}{\lambda}\sum_{i=0}^{\infty}(-1)^i\binom{ph+p-i}{i}(a-1)^i B(i+1, ph+pm+p)$$
$$= \frac{e^{-w(1-pm)}}{\lambda}\sum_{i=0}^{\infty}(-1)^i\binom{ph+p-i}{i}(e^{-w}-1)^i B(i+1, ph+pm+p). \quad (3.9)$$

We substitute for Λ_1 in equation (3.8), we have

$$\Lambda = p\sum_{h=0}^{r+j}\sum_{m=0}^{n-2r+k-j+l-1}\sum_{i=0}^{\infty}(-1)^{h+m+i}\binom{r+j}{h}\binom{n-2r+k-j+l-1}{m}\binom{ph+p-i}{i}$$
$$\times B(i+1, ph+pm+p)e^{-w}(e^{-w}-1)^i. \quad (3.10)$$

Finally,

$$P_r(W \le w) = \sum_{k=0}^{r}\prod_{i=0}^{2r-k}\frac{(n-i)}{r!\,(r-k)!}\sum_{j=0}^{n-2r+k-1}(-1)^j\binom{n-2r+k-1}{j}\sum_{l=0}^{r-k}(-1)^l\binom{r-k}{l}$$
$$\times p\sum_{h=0}^{r+j}\sum_{m=0}^{n-2r+k-j+l-1}\sum_{i=0}^{\infty}(-1)^{h+m+i}\binom{r+j}{h}\binom{n-2r+k-j+l-1}{m}\binom{ph+p-i}{i}$$
$$\times B(i+1, ph+pm+p)e^{-w}(e^{-w}-1)^i. \quad (3.11)$$

It should be noted also that the distribution function of quasi-range of the negatively skewed extended generalized logistic distribution is independent of parameter λ but depends on p.

4. Covariance between the m^{th} and the q^{th} order statistics from the negatively skewed extended generalized logistic distribution

The joint moment generating function of $X_{m:n}$ and $X_{q:n}$ where $m > q$ and n is the sample size is

$$M(t_1, t_2) = E[\exp(xt_1 + yt_2)]$$
$$= C\int_{-\infty}^{\infty}dy\int_{-\infty}^{y}\exp(xt_1 + yt_2)\,F^{q-1}(x)[F(y)-F(x)]^{m-q-1}[1-F(y)]^{n-m}f(x)f(y)dx, \quad (4.1)$$

where $C = n!/[(q-1)!\,(m-q-1)!\,(n-m)!]$, $f(x) = f_X(x;\lambda,p)$ and $F(x) = F_X(x;\lambda,p)$ as given in equations (1.3) and (1.4) respectively. Let $p_1 = [1-(e^{-x}/(\lambda+e^{-x}))^p]$, $p_2 = [1-(e^{-y}/(\lambda+e^{-y}))^p]$, then $\exp(xt_1) = ((1-\sqrt[p]{1-p_1})/(\lambda\sqrt[p]{1-p_1})))^{t_1}$ and $\exp(yt_2) = ((1-\sqrt[p]{1-p_2})/(\lambda\sqrt[p]{1-p_2})))^{t_2}$, $dx = (\lambda+e^{-x})^{p+1}(p\lambda e^{-px})^{-1}dp_1$ and $dy = (\lambda+e^{-y})^{p+1}(p\lambda e^{-py})^{-1}dp_2$. So, by substituting all these into equation (4.1), we have

$$M(t_1, t_2) = C\int_0^1\left(\frac{1-\sqrt[p]{1-p_2})}{\lambda(\sqrt[p]{1-p_2})}\right)^{t_2}(1-p_2)^{n-m}dp_2\int_0^{p_2}\left(\frac{1-\sqrt[p]{1-p_1})}{\lambda(\sqrt[p]{1-p_1})}\right)^{t_1}p_1^{q-1}(p_2 - p_1)^{m-q-1}dp_1 \quad (4.2)$$
$$= \frac{C}{\lambda^{t_1+t_2}}\sum_{i=0}^{m-q-1}(-1)^i\binom{m-q-1}{i}\int_0^1\left(\frac{1-\sqrt[p]{1-p_2})}{(\sqrt[p]{1-p_2})}\right)^{t_2}(1-p_2)^{n-m}p_2^i dp_2$$
$$\times\int_0^{p_2}\left(\frac{1-\sqrt[p]{1-p_1})}{(\sqrt[p]{1-p_1})}\right)^{t_1}p_1{}^{m-i-2}dp_1. \quad (4.3)$$

Let $H_1 = \int_0^{p_2} \left((1-p_1)^{-\frac{1}{p}} - 1\right)^{t_1} {p_1}^{m-i-2} dp_1$

$$= \int_0^{p_2} \sum_{h=0}^{t_1} (-1)^h \binom{t_1}{h} (1-p_1)^{-h/p} {p_1}^{m-i-2} dp_1$$

$$= \sum_{h=0}^{t_1} (-1)^h \binom{t_1}{h} \int_0^{p_2} \sum_{j=0}^{\infty} (-1)^j \binom{h/p+j-1}{j} {p_1}^{j+m-i-2} dp_1$$

$$= \sum_{h=0}^{t_1} \sum_{j=0}^{\infty} (-1)^{h+j} \binom{t_1}{h} \binom{h/p+j-1}{j} \frac{p_2^{j+m-i-1}}{j+m-i-1}. \qquad (4.4)$$

We put expression (4.4) in (4.3) to obtain

$$M(t_1,t_2) = \frac{C\lambda^{-(t_1+t_2)}}{m+j-i-1} \sum_{i=0}^{m-q-1} \sum_{h=0}^{t_1} \sum_{j=0}^{\infty} (-1)^{i+h+j} \binom{m-q-1}{i} \binom{t_1}{h} \binom{h/p+j-1}{j}$$
$$\times \int_0^1 \left(\frac{1-\sqrt[p]{1-p_2})}{(\sqrt[p]{1-p_2})}\right)^{t_2} (1-p_2)^{n-m} {p_2}^{j+m-1} dp_2. \qquad (4.5)$$

Let

$$H_2 = \int_0^1 \left(\frac{1-\sqrt[p]{1-p_2})}{(\sqrt[p]{1-p_2})}\right)^{t_2} (1-p_2)^{n-m} {p_2}^{j+m-1} dp_2$$

$$= \int_0^1 (1-p_2)^{n-m} {p_2}^{j+m-1} ((1-p_2)^{-1/p} - 1)^{t_2} dp_2$$

$$= \sum_{k=0}^{t_2} (-1)^k \binom{t_2}{k} \int_0^1 (1-p_2)^{n-m-k/p} {p_2}^{j+m-1} dp_2$$

$$= \sum_{k=0}^{t_2} (-1)^k \binom{t_2}{k} B(j+m, n-m-k/p). \qquad (4.6)$$

Now we put (4.6) in (4.5) and substitute for C to obtain

$$M(t_1,t_2) = \sum_{i=0}^{m-q-1} \sum_{h=0}^{t_1} \sum_{j=0}^{\infty} \sum_{k=0}^{t_2} (-1)^{i+h+j+k} \binom{m-q-1}{i}$$
$$\times \binom{t_1}{h} \binom{h/p+j-1}{j} \binom{t_2}{k} \frac{n!\, \lambda^{-(t_1+t_2)} B(j+m, n-m-k/p)}{(m+j-i-1)(q-1)!\,(m-q-1)!\,(n-m)!}, \qquad (4.7)$$

From the above expression (4.7) which is the joint moment generating function of the m^{th} and q^{th} order statistics from the negatively skewed extended generalized logistic distribution, one can obtain the r^{th} and s^{th} bivariate moment as

$$E\left(X^r{}_{q:n} X^s{}_{m:n}\right) = \frac{\partial^{r+s}}{\partial t_1^r \partial t_2^s} M(t_1,t_2)|_{t_1=t_2=0.} \qquad (4.8)$$

5. Conclusions

The distribution functions of the range and quasi-range of the negatively skewed extended generalized logistic distribution have been established. The covariance between the m^{th} and the q^{th} order statistics has been obtained.

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