American Review of Mathematics and Statistics
June 2015, Vol. 3, No. 1, pp. 88-105
ISSN: 2374-2348 (Print), 2374-2356 (Online)

Published by American Research Institute for Policy Development
DOI: 10.15640/arms.v3n1a9
URL: http://dx.doi.org/10.15640/arms.v3n1a9

Does the Sequences of Random Numbers Follows a Log Periodic Law? Randomness has Memory?

David Izabel[1]

Abstract

The evolution law found by L. Nottale, J.Chaline and P. Grou [13] allows to predict with success the evolution in the time of biological phenomena… In this paper, we show that after a light transposition, it allows to predict also in sequences of decimals like π or φ the gold number or e the Euler number, the mathematic evolution of the cumulative number of new numerals that appears draw after draw by comparison with a reference draw taken arbitrary. It allows equally, in a series of draws of game of chance like the Loto, to predict draw after draw the cumulative number of new numbers released by comparison with a reference draw taken arbitrary in the list of consecutive draws. Of the evolution of this cumulative number of new numerals obtained at each successive draw, we obtain the cumulative number of old numerals and the probability/proportion (cumulative and at each draw) of the new and old numeral sat each draw. Two theorems are proposed at the end of this paper to conclude this research.

Keywords: Randomness memory, fractal, analysis of sequence of decimals of transcendent number, game theory

I. Introduction

The mathematics progression or sequences are several shapes. We have firstly the arithmetic sequences that have the following shape:

$u_{n+1} = u_n + r \quad (1)$;

Secondly, the geometric progression that have the following shape: $u_{n+1} = q \times u_n \quad (2)$ with $u_0 = a$;

And finally, the arithmetic-geometric sequences, topic of this paper that have the following shape:

$u_{n+1} = au_n + b \quad (3)\ \forall n \in$ N.

After this reminder, we consider now a game of chance (randomness game like the Loto for example) that have N numbered balls between 1 and N. At each draw T_i the n numbered balls are all put again in the game. During a draw n numerals are drawn. We consider also a successive draw sequence taken into account the time that all the possible numbers N of the game appear all one time at minimum. We suppose that, with an appropriate law, we arrive to predict the cumulative numbers of new numerals (new numbered balls) noted C_i that appears at each new draw T_i by comparison with a referential draw noted T_{ref} taken arbitrary as start point in a list of successive draws. In this case, we can estimate P_i the probability/proportion of old numerals at each successive draw T_i by comparison with referential draw T_{ref}.

[1] Engineer and Technical Manager 106 rue Lemercier 75017 Paris (France). Phone: 00 33 1 42 94 23 01.
Email: d.izabel@aliceadsl.fr

The probability/proportion p_{i+1} of new balls at the draw T_{i+1} takes the following expression:

$$p_{i+1} = \left(\frac{C_{i+1} - C_i}{n}\right) \quad (4)$$

We deduct of (4) the probability/proportion of old numerals P_{i+1}arriving at the draw T_{i+1} by the following expression:

$$P_{i+1} = 1 - \left(\frac{C_{i+1} - C_i}{n}\right) \quad (5)$$

From the expression (5) we can obtain the cumulative number of new balls at each draw T_i by comparison with the reference draw via an arithmetic-geometric sequence that have the following shape.

$$C_{i+1} = C_i + n(1 - P_{i+1}) \quad (6)$$

We can see that this expression (6) is very near of the mathematic frame of the arithmetic-geometric sequence given in the expression (3). The aim of this paper is to study how to estimate C_i the cumulative number of new numerals arriving at the draw T_i via a fractal law. We will see how from this number C_i deduct the probability/proportion P_i of old number at each draw. We will apply the previous approach and formula at game of chance like the Loto, Keno but also via a simple statistic approach (similar way at the Benford law) at decimal sequences of transcendent number π , the gold number φ (1.618) or the Euler number e (2.718).

II. The Evolution Law

The evolution theory or law J. Nottale, J. Chaline and P. Grou have been published in 1999 in the Eureka review[13]and in lot of other articles[1],[2],[4],[6],[7].This law is based on the works about the periodicity log of D. Sornette[5],[8] [9],[10],[11],[12]. This law follows the fractal theory. For memory, a fractal figure is a curve or an area with a not regular shape that follows determinist and stochastic rule directly in link with an internal homothetic transformation. The sheet of the fern for example is a fractal object (see fig 1).

Figure 1: Typical Fractal Aspect of the Fern

The evolution law takes the following shape:

$$T_n = T_c + (T_0 - T_c)g^{-n} \quad (7)$$

Or with the logarithm function:

$$\log|T_n - T_c| = \log|T_0 - T_c|T_c - n\log(g) \quad (8)$$

The formula (8) is periodic log, this explain the title of this paper.

These formulations have been applied at the time variable T to explain the evolution of several biological species or to analysis important events during a time T_c (index C for critical).

In the expression (8):

T_n : is representing the mutation date of rank n

n: Figure the rank of the event

T_c : The critical time of the scalable sequences (end in the time of a period of a cycle),

T_0 : The date of rank 0,

$g = k^{1/D}$ (9) : Relationship between 2 consecutive branches(cf. figure 2 and see[14]and[15]for the source of this formula,

Log: Logarithm in base 10,

$k = g^D$ (9bis) : An integer, if the number of consecutive branch stay constant at each step (cf. figure 2),

D :A digital exposing that makes a link between the scale factor and the branch number and the origin of the evolution considered, determined for an event arbitrary chosen at n = 0.

D is a fractal dimension (cf. figure 2) that characterize successive bifurcation of the branches,

Example of application:

For a tree the conservation of the flaw of sap threw the trunk and its branches (with a radius r) is written as follow:

$k\, r_{n+1}^2 = r_n^2$ And on a more general way with a fractal dimension D: $k\, r_{n+1}^D = r_n^D$

The tree is auto-similar by hypothesis we suppose that g don't change for each scalable sequences. In function of the ratio of the consecutive branch or in function of the time, g takes the expression (10).

$$g = \frac{T_{n+1} - T_n}{T_{n+2} - T_{n+1}} = \frac{T_n - T_C}{T_{n+1} - T_C} = \frac{L_n}{L_{n+1}} = \frac{T_n - T_{n-1}}{T_{n+1} - T_n} \quad (10)$$

L_n: length of the tree branch between the times $T_{n-1}$and T_n

This law have been applied on the temporal variable T or on more complex variables (space time).

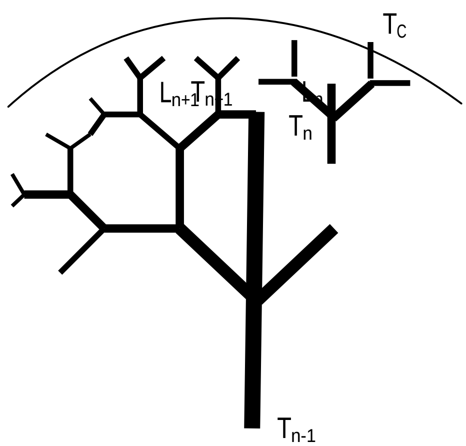


Figure 2: Fractal Representation Of G

So, this theory allows to quantify the dates where the nature changes of direction, where there is an evolution. This law must keep the same evolution law regardless of the number of necessary branches. We are going to transpose now this law at a series of random numbers that can be the decimals of the numberπ(3.14…), the Euler number e (2.718…), or the gold numberφ (1.618…), or a series of draws (game heads or tails, craps ,Loto, Kéno (Française des jeux) etc.The duration of the series is corresponding to the necessary time for that all the possible numbers (named N) appears at minimum one time. A new series begin when all the possible numbers are appeared in the series of successive draws. The study of the law that define the evolution of the apparition of new numbers during the successive draws is one of the objective of this paper.

III Transposition of the L. Notale, J. Chaline and P. Grou law at Decimals Sequences of Random Game (Game of Chance)

III-1 General Theory

We laid as hypothesis that the evolution of the cumulative number of new numerals appearing at each draw evolve following a fractal law. The time of a cycle correspond at the time necessary for that all the possible numbers of the game or of the series of decimals appears one time at minimum. This approach is similar to the branches of a tree (CF III-2). A cycle is finished when in a decimals series or in a series of draw of a game of chance, all the different and possible numerals of the game are all appears one time at minimum.

The expression (7) becomes:

$$C_i = C_C + (C_0 - C_C)g^{-Ti} \quad (11)$$

In this expression:

Is the rank number of the draw by comparison with the referential draw. itakes the value 1 for the first rank ; i = c for the last rank.

If T is representing the group of T_i, the numbers of successive draws of the game taken from a referential draw taken arbitrary in a list of successive draws as defined below:

$$1 \le T_i \le T_c \ \forall i \in [0; c]$$

- T_0 (i = 0) is the virtual draw (not real) before the first draw. It correspond at 0 number drawn
- T_1 (i = 1) correspond at the n first new numbers or numerals obtain during the first draw,
- T_C (i = c) correspond at the last number or numeral drawn possible obtain at the last draw of the series considered.

Are supposed known:

- C_0: The value of the cumulative number of new numerals drawn for the first time before the first draw$T_i = T_0$;so it is 0 numerals (virtual draw).
- C_1: The value of the cumulative number of numerals drawn for the first time at the first draw; so it is the n first new numbers (or numerals) of the game: $C_1 = n$ with 1 ≤ n ≤N. We suppose that the number of numerals drawn at each draw n = constant.
- C_c: The value of the cumulative number of new numerals drawn for the first time at the last draw; so it is the numbers of different numerals that are possible in the game or in the decimals series: N : $C_c = N$ Avec N ≥ 1

So we are looking for:

- C_i: The value of the cumulative number of numerals drawn for the first time at the draw T_i calculated from the referential draw taken arbitrary in a series of consecutive draws of a game of chance or decimals series of a transcendant number

An estimation of the parameter g for each type of game is possible when the relationship between two consecutive events follows the formula (10):

So we have: $g = \frac{C_0 - C_c}{C_1 - C_c} = \frac{0 - N}{n - N} = \frac{N}{N - n} \quad (12)$

The evolution law of L.Nottale, J. Chaline and P. Groutransposed to the random game (game of chance) given in (11) becomes with g obtained from the expression (12):

$$C_i = N + (0 - N)\left[-\frac{N}{n - N}\right]^{-T_i}$$

So:

$$\boxed{C_i = N - N\left[\frac{N}{N - n}\right]^{-T_i} = N\left\{1 - \frac{1}{g^{Ti}}\right\} \quad (13)}$$

The table 1 below gives a summarize of the relationship between the notations of L. Nottale, J. Chaline and P. Grouthe notations used to predict the evolution/proportion of the new numerals at each draw T_i until that all the possible numbers or numerals of the game or of the decimals series are appeared.

Habitual notations used in the law of Nottale Chaline and Grou	**Application at the sequences of decimals**	**Characteristic Values**
Critical time T_c	C_c = N The maximum number of different decimals or different possible numbers in a game of chance	N = 2 heads or tails game N = 6 set of one dice N = 10 (0 à 9) for π or φ or e N = 49 for theLoto N = 70 for theKéno
Time of rank 0 T_0	C_0 = 0 the cumulative number of new numerals before the first rank	Its value is 0 systematically
Time of rank 1 T_1	C_1 = n the maximum numberof new numerals at the first draw T_1.	n = 1 heads or tails game n = 1 set of one dice n = 5 for the Loto (7 for the old formula) n = 10 for the decimals sequences of transcendant number like π, φ or e n = 20 for the Kéno
Time of rank n T_n	C_i = The cumulative number of new numbers at the ranki(draw i)	The analysis of the random numbers is done on a series of successive draws (from T_1 to T_c) the time that all the possible numerals of the game of chance (N) or of the sequences of decimals appears all at minimum one time.
g fractal ratio $g=\frac{T_n-T_c}{T_{n+1}-T_c}$	$g=\frac{C_0-C_c}{C_1-C_c}=\frac{0-N}{n-N}$	g = 1.11 for a decimal series g = 1.1136 for the Loto (5 numeral balls) g = 1.166 for the Loto (7 numeral balls) g = 1.2 for the set of dice g = 1.4 for the Kéno g = 2.0 for the heads and tails game
$T_n=T_c+(T_0-T_c)g^{n}$	$C_i=C_c+(C_0-C_c)g^{-Ti}$ $C_i=N-N\left(\frac{N}{N-n}\right)^{-Ti}$	The cumulative number of new numerals or at the draw T_i on a series of successive draw taken between T_1 until T_c

Table 1: Comparison of the Notations and Signification of the Different Parameters of the Evolution Law

III-2 Illustration of the Fractal Aspect of a Game of Chance

For the game heads and tails, an illustration of the associated fractal tree is given at the figures 7 and 8. The ratio of the number of branch to the previous branches k takes the value of 2 at each roll of the coin (either head or tail). The dimensions and characteristic of the branch doesn't change at each roll; this imply D =1.

$g=k^{1/D} \Rightarrow g=2^{1/1}=2$

IV Application of the Evolution law Transposed at the Sequences of Random Decimals for the Calculation of C_i

IV-1 Example of Determination of the C_i for the First Decimals of the Transcendant Number π

The definition of the parameters is the following:

N = 10; n = 1; g = 1.1111111

The first values of the C_i are the following:

Before all draw: $C_0=0$

At the draw number 1: $C_1=1$

At the draw number 2: $C_2=10-10\left(\frac{10}{10-1}\right)^{-2}=1.9$

At the draw number 3: $C_3 = 10 - 10\left(\dfrac{10}{10-1}\right)^{-3} = 2.71$

The analysis of the first decimals is given in the table 2

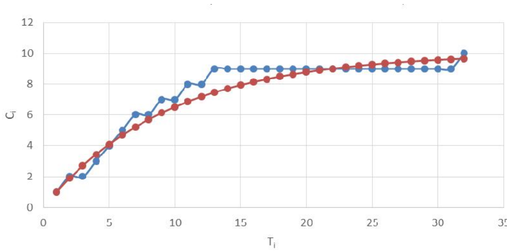
The color code is the following:

Yellow: identification of the draw corresponding at the apparition of the number;
Blew: correspond the apparition of the number for the first time;
Green: number already drawn;

NbN: Number of new balls at each drawn.

3,	rang Ti	0	1	2	3	4	5	6	7	8	9	NbN	Ci	N° tirage	Ci mesuré	Ci th evolution
1	1											1	1	1	1	1
4	2											1	2	2	2	1,9
1	3											0	2	3	2	2,71
5	4											1	3	4	3	3,439
9	5											1	4	5	4	4,0951
2	6											1	5	6	5	4,68559
6	7											1	6	7	6	5,217031
5	8											0	6	8	6	5,6953279
3	9											1	7	9	7	6,12579511
5	10											0	7	10	7	6,513215599
8	11											1	8	11	8	6,861894039
9	12											0	8	12	8	7,175704635
7	13											1	9	13	9	7,458134172
9	14											0	9	14	9	7,712320755
3	15											0	9	15	9	7,941088679
2	16											0	9	16	9	8,146979811
3	17											0	9	17	9	8,33228183
8	18											0	9	18	9	8,499053647
4	19											0	9	19	9	8,649148282
6	20											0	9	20	9	8,784233454
2	21											0	9	21	9	8,905810109
6	22											0	9	22	9	9,015229098
4	23											0	9	23	9	9,113706188
3	24											0	9	24	9	9,202335569
3	25											0	9	25	9	9,282102012
8	26											0	9	26	9	9,353891811
3	27											0	9	27	9	9,41850263
2	28											0	9	28	9	9,476652367
7	29											0	9	29	9	9,52898713
9	30											0	9	30	9	9,576088417
5	31											0	9	31	9	9,618479576
0	32											1	10	32	10	9,656631618

Table 2: List of the 32 first Decimals of the Number π, – n° of rank T_i – Determination of the Number of New Decimals at each draw – Table of C_i Measured and Theoretical



C_i measured C_i following Theory of evolution transposed at the list of decimals

Figure 3: Comparison of the C_i Theoretical and Measured for the First Series of Decimals of the Number π

IV-2 Example of Determination of the C_i of the first Decimals of the Gold Number φ

The definition of the parameter is the following:

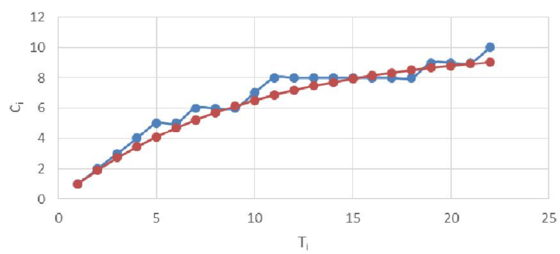
N = 10; n = 1; g = 1.1111111

The analysis of the first decimals is given in the table 3:

1,	rang Ti	0	1	2	3	4	5	6	7	8	9	NbN	Ci
6	1											1	1
1	2											1	2
8	3											1	3
0	4											1	4
3	5											1	5
3	6											0	5
9	7											1	6
8	8											0	6
8	9											0	6
7	10											1	7
4	11											1	8
9	12											0	8
8	13											0	8
9	14											0	8
4	15											0	8
8	16											0	8
4	17											0	8
8	18											0	8
2	19											1	9
0	20											0	9
4	21											0	9
5	22	0	1	2	3	4	5	6	7	8	9	1	10

N° tirage	Ci mesuré	Ci th évolution
1	1	1
2	2	1,9
3	3	2,71
4	4	3,439
5	5	4,0951
6	5	4,68559
7	6	5,217031
8	6	5,6953279
9	6	6,12579511
10	7	6,513215599
11	8	6,861894039
12	8	7,175704635
13	8	7,458134172
14	8	7,712320755
15	8	7,941088679
16	8	8,146979811
17	8	8,33228183
18	8	8,499053647
19	9	8,649148282
20	9	8,784233454
21	9	8,905810109
22	10	9,015229098

Table 3: List of the 22 First Decimals of the gold number φ, – n° de rank T_i – Determination of the Number of New Decimals at each draw– Table of C_i Measured and Theoretical



C_i measured C_i following Theory of evolution transposed at the list of decimals

Figure 4: Comparison of the C_i theoretical and Measured for the first Series of Decimals of φ

IV-3 Example of Determination of the C_i for 44 successive draws of the game Loto

The definition of the parameters is the following:

N = 49; n = 5; g = 1.11363636

The first values of the constant C_i are the following:

Before any draw: $C_0 = 0$

At the draw number1: $C_1 = 5$

At the draw number 2: $C_2 = 49 - 49\left(\dfrac{49}{49-5}\right)^{-2} = 9.48979592$

At the draw number 3: $C_3 = 49 - 49\left(\dfrac{49}{49-5}\right)^{-3} = 13.5214494$

The table 4 after gives the results (5 numerals drawn by draw) of 44 successive draws computed from a reference draw taken arbitrary in a list of successive draws obtained on the net.

N° de Tirage	x1(Ti)	x2(Ti)	x3(Ti)	x4(Ti)	x5(Ti)
1	16	22	38	42	47
2	4	10	13	20	38
3	3	13	14	15	42
4	1	17	40	42	43
5	6	16	26	31	40
6	7	23	27	28	33
7	2	10	16	35	44
8	3	6	10	18	23
9	22	25	31	34	46
10	1	13	26	31	44
11	10	16	25	38	49
12	9	26	29	35	38
13	4	17	28	33	41
14	16	17	27	42	44
15	12	14	29	41	45
16	9	20	22	40	46
17	4	13	21	23	43
18	10	15	19	24	28
19	1	12	15	29	30
20	9	28	37	41	46
21	1	15	24	27	43
22	13	21	26	30	40
23	10	19	33	35	47
24	14	24	39	47	49
25	12	13	28	32	33
26	6	17	18	29	32
27	2	9	29	36	48
28	18	24	34	44	45
29	1	24	27	37	46
30	7	14	27	40	46
31	6	18	23	24	30
32	13	19	23	27	40
33	3	16	23	28	39
34	16	23	35	47	48
35	13	14	18	29	41
36	1	9	22	26	44
37	17	18	32	47	49
38	5	12	20	34	47
39	2	18	44	47	49
40	40	43	44	48	49
41	11	26	28	31	49
42	10	18	20	37	43
43	10	14	32	43	47
44	3	8	11	16	27

Table 4: Data list of 44 Successive draws of the game Loto

The analysis of these 44 draws in view to determine C_i at each draw is given in the table 5 just after. The comparison between the measured C_i and the theoretical C_i obtain with the transposition of the evolution law is given at the table 6 and at the figure 5. We can observe an excellent correlation between the two curves for this series of values.

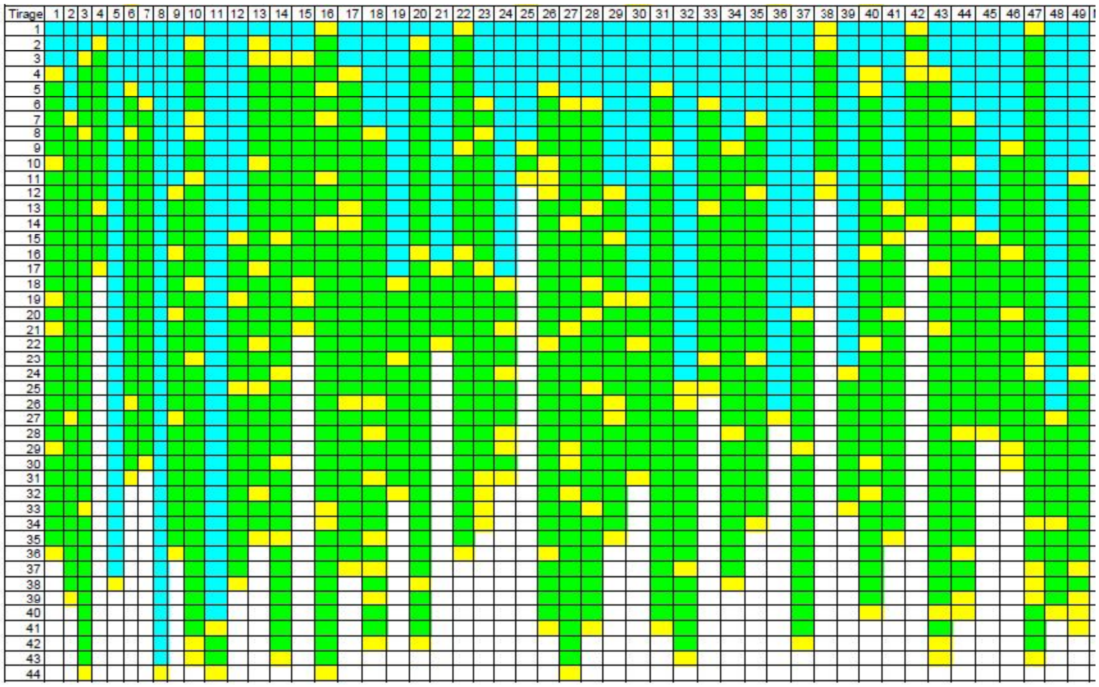
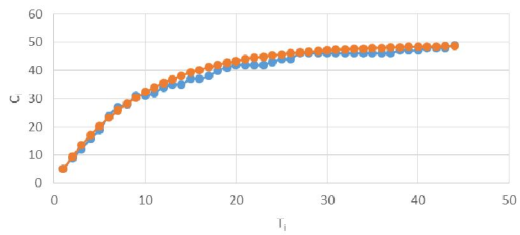


Table 5: Data List of the 44 Successive draws of the Loto (End of 2014)

N° de Tirage	Ci mesuré	Ci Th évolution
1	5	5
2	9	9,489795918
3	12	13,5214494
4	16	17,14170966
5	19	20,39255561
6	24	23,31168259
7	27	25,93293947
8	28	28,28672116
9	31	30,40032104
10	31	32,29824746
11	32	34,00250793
12	34	35,53286426
13	35	36,90706179
14	35	38,14103507
15	37	39,24909272
16	37	40,24408326
17	38	41,13754415
18	40	41,93983556
19	41	42,66026051
20	42	43,3071727
21	42	43,88807344
22	42	44,4096986
23	42	44,8780967
24	43	45,29869908
25	44	45,67638285
26	44	46,01552746
27	46	46,32006547
28	46	46,59352818
29	46	46,83908653
30	46	47,0595879
31	46	47,25758914
32	46	47,43538616
33	46	47,59504064
34	46	47,73840384
35	46	47,86713814
36	46	47,98273629
37	46	48,08653871
38	47	48,17974904
39	47	48,26344812
40	47	48,33860648
41	48	48,40609561
42	48	48,4666981
43	48	48,52111666
44	49	48,56998231

Table 6: $C_i$measured and Theoretical game (Loto



C_i measured C_i following Theory of evolution transposed at the game

Figure 5: Comparison of the C_i Theoretical and Measured for 44 Successive draws of the Game of chance Loto

IV-4 Example of Determination of the C_i for 14 Successive draw of the Game of chance Kéno

The definition of the parameters is the following:

N = 70; n = 20; g = 1.4

The first values of the C_i are the following:

Before any draw: $C_0 = 0$

At the draw number 1: $C_1 = 1$

At the draw number 2: $C_2 = 70 - 70\left(\frac{70}{70-20}\right)^{-2} = 34.2857$

At the draw number 3: $C_3 = 10 - 10\left(\frac{10}{10-1}\right)^{-3} = 44.489$

The tables 7 and 7 bisjust after gives the results of14 successive draws (20 numbers by draw) computed from a reference draw taken arbitrary in a list of successive draws free on the net for the game Keno.

Ti	x1(Ti)	x2(Ti)	x3(Ti)	x4(Ti)	x5(Ti)	x6(Ti)	x7(Ti)	x8(Ti)	x9(Ti)	x10(Ti)
1	5	7	8	11	14	17	31	32	33	35
2	8	11	13	16	17	23	25	26	32	39
3	6	7	9	10	15	16	22	28	37	38
4	2	7	9	18	20	22	24	25	29	36
5	2	5	11	13	28	32	33	35	37	40
6	1	3	9	13	14	15	21	29	32	35
7	1	3	7	8	10	12	17	18	25	33
8	1	6	18	19	24	33	34	37	38	42
9	2	5	9	10	11	12	20	23	28	29
10	1	12	13	14	21	25	26	32	36	37
11	4	8	13	14	16	18	21	22	23	31
12	2	3	5	6	10	11	15	22	28	29
13	1	10	12	13	19	23	26	30	31	34
14	3	4	5	11	14	16	19	20	25	27

Table 7: Data list of 14 Successive draws of the game of Chance Kéno

Ti	x11(Ti)	x12(Ti)	x13(Ti)	x14(Ti)	x15(Ti)	x16(Ti)	x17(Ti)	x18(Ti)	x19(Ti)	x20(Ti)
1	36	37	48	51	53	57	60	63	67	68
2	51	52	53	55	56	57	59	67	68	70
3	39	46	50	54	57	60	61	63	65	67
4	41	46	55	59	60	61	63	64	65	67
5	42	51	52	55	58	60	62	65	68	70
6	37	38	39	41	50	52	53	61	63	70
7	37	42	43	48	55	56	58	63	65	68
8	44	48	49	50	51	53	59	64	65	67
9	30	33	42	47	59	60	65	66	68	69
10	39	45	52	55	58	59	63	64	69	70
11	32	36	40	41	45	49	52	58	59	69
12	39	45	48	50	51	56	62	63	65	66
13	36	44	52	53	55	56	57	60	64	65
14	28	29	35	36	48	51	57	58	67	68

Table 7bis: Data list of 14 Successive draws of the game of Chance Kéno

The analysis of the 14 successive draws in view to determine the $C_i$at each drawn is given in the table 8 just after. The comparison between the measured C_i and theoretical C_i tis given at the table 9 and at the figure 6. We can see an excellent correlation between the two series of points of the two curves

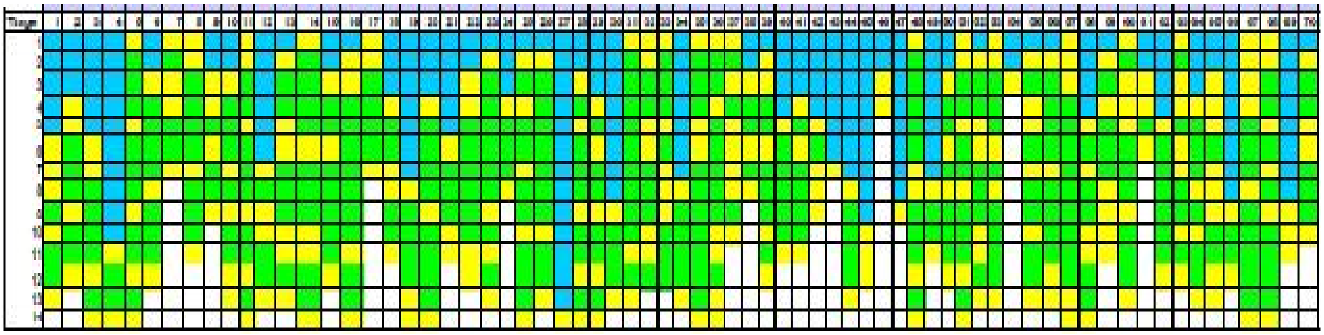
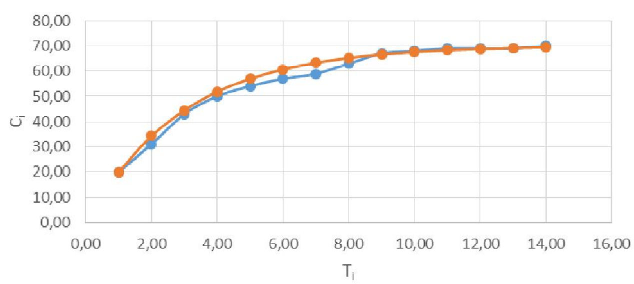


Table 8: Data List of 14 Successive draws of the Game of chanceKéno

N°tirage	Ci mesuré	Ci Th évolution
1,00	20,00	20
2,00	31,00	34,28571429
3,00	43,00	44,48979592
4,00	50,00	51,77842566
5,00	54,00	56,98458975
6,00	57,00	60,7032784
7,00	59,00	63,35948457
8,00	63,00	65,25677469
9,00	67,00	66,61198192
10,00	68,00	67,57998709
11,00	69,00	68,27141935
12,00	69,00	68,76529953
13,00	69,00	69,1180711
14,00	70,00	69,37005078

Table 9: Measured and Theoretical C_i for the game of Chance (Kéno)



C_i measured C_i following Theory of evolution transposed at the game

Figure 6: Comparison of Theoretical and Measured C_i for 14 Successive draws of the game Kéno

V. Application of the Evolution law Transposed at the Random Sequences of Decimals at the Calculation of the Probability P_i

V-1 Development of the Expression of the Probability P_i of Apparition of old Number (already drawn) in Function of the Data of the Problem

The probability/proportion of old numbers (= already drawn) at the draw $T_{i+1}$by comparison with the referential draw takes the following expression:

$$P_{i+1} = 1 - \left(\frac{C_{i+1} - C_i}{n}\right) \quad (5) \quad \text{With} \quad C_i = N - N\left[\frac{N}{N-n}\right]^{-T_i} \quad (13)$$

If we put the expression (13) of C_i in the expression (5) of P_{i+1} we obtain:

$$P_{i+1} = \frac{n - N\left\{\left(-\left[\frac{N}{N-n}\right]^{-Ti+1} + \left[\frac{N}{N-n}\right]^{-Ti}\right)\right\}}{n} \quad P_i = \frac{n - N\left\{\left(-\left[\frac{N}{N-n}\right]^{-Ti} + \left[\frac{N}{N-n}\right]^{-Ti-1}\right)\right\}}{n} \quad \boxed{P_i = \frac{n - N\left\{\left([g]^{-Ti-1} - [g]^{-Ti}\right)\right\}}{n} \quad (14)}$$

$$P_i = \frac{n}{n} - \frac{N}{n}\left[\left(\frac{N-n}{N}\right)^{Ti-1} - \left(\frac{N-n}{N}\right)^{Ti}\right]$$

$$P_i = 1 - \frac{N}{n}\left[\frac{(N-n)^{Ti-1}}{N^{Ti-1}} - \frac{(N-n)^{Ti-1}(N-n)}{N^{Ti-1}N}\right] \quad P_i = 1 - \frac{N}{n}\left[\frac{N(N-n)^{Ti-1}}{NN^{Ti-1}} - \frac{(N-n)^{Ti-1}(N-n)}{N^{Ti-1}N}\right] \quad P_i = 1 - \frac{N(N-n)^{Ti-1}}{nN^{Ti-1}N}[N-(N-n)]$$

$$\boxed{P_i = 1 - \frac{(N-n)^{Ti-1}}{N^{Ti-1}} = 1 - \frac{N(N-n)^{Ti}}{N^{Ti}(N-n)} = \frac{N^{Ti} - N(N-n)^{Ti-1}}{N^{Ti}} \quad (15)}$$

V-2 Example of Determination of P_i for the Game Heads and Tails

The following question is given: At the second roll of coin, what is the probability to obtain the same results that at the first roll? The answer is 2 possibilities on 4 so a probability of 0.5 as indicated at the figure 7 just after.

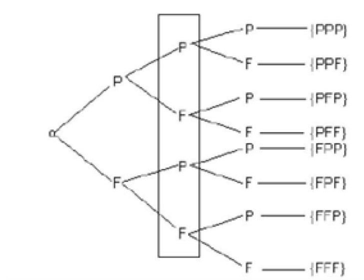


Figure 7: Game Heads and Tails – Probability to obtain at the Second roll the same Result that at the First Roll

If we use the evolution law calibrated for the game, we obtain:

N = 2, n =1, g = 2

$P_i = \frac{n - N\{([g]^{-Ti-1} - [g]^{-Ti})\}}{n}$ so $P_2 = \frac{1 - 2\{2^{-1} - 2^{-2}\}}{1} = 0.5$

The following question is now asked: At the third roll of the coin, what is the probability to obtain a result obtain at the two previous roll? The answer is 6 possibilities on 8 so a probability of 0.75 (see figure 8 below).

If we use the evolution law calibrated at the game, we obtain:

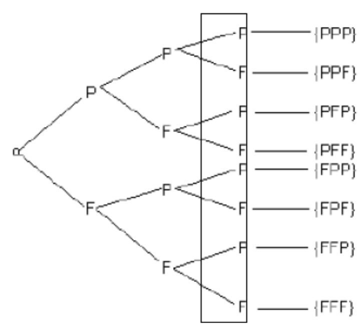


Figure 8: Game Heads and Tails – Probability to obtain at the Third roll a Result already obtained at the Previous Roll

$P_i = \frac{n - N\{([g]^{-Ti-1} - [g]^{-Ti})\}}{n}$ so $P_3 = \frac{1 - 2\{2^{-2} - 2^{-3}\}}{1} = 0.75$

We verify so well, that in the case of the game heads and tails, the probability/proportion of repetition of a previous results at the draw T_i follow the formula (15) $P_i = \frac{N^{Ti} - N(N-n)^{Ti-1}}{N^{Ti}}$

Additionally, as N-n =1 for this game we have:

$P_i = \frac{N^{Ti} - N}{N^{Ti}}$ so: $P_1 = \frac{2^1 - 2}{2^1} = 0$; $P_2 = \frac{2^2 - 2}{2^2} = 0.5$; $P_3 = \frac{2^3 - 2}{2^3} = \frac{6}{8} = 0.75$;...*etc*

V-3 Example of Determination of the Probability P_i in the case of the Dice Game

The following question is asked: At each roll of dice, what is the probability to obtain a numeral already drawn at the previous draw?

If we use the evolution law calibrated at the dice game, we obtain:

N = 6, n =1, g = 1.2

At the draw number from the formula (14):

$P_i = \frac{n - N\{([g]^{-Ti-1} - [g]^{-Ti})\}}{n}$ so $P_1 = \frac{1-6\{1.2^{0} - 1.2^{-1}\}}{1} = 0$

At the draw number 2:

$$P_2 = \frac{1-6\{1.2^{-1} - 1.2^{-2}\}}{1} = 0.1666666$$

At the draw number 3:

$$P_3 = \frac{1-6\{1.2^{-2} - 1.2^{-3}\}}{1} = 0.305555$$

At the draw number 4:

$$P_4 = \frac{1-6\{1.2^{-3} - 1.2^{-4}\}}{1} = 0.421296296$$

The verification of these results is given at the figure 9. The yellow cases below indicate the good cases of the numerals that are repeated by comparison with the numerals already drawn.

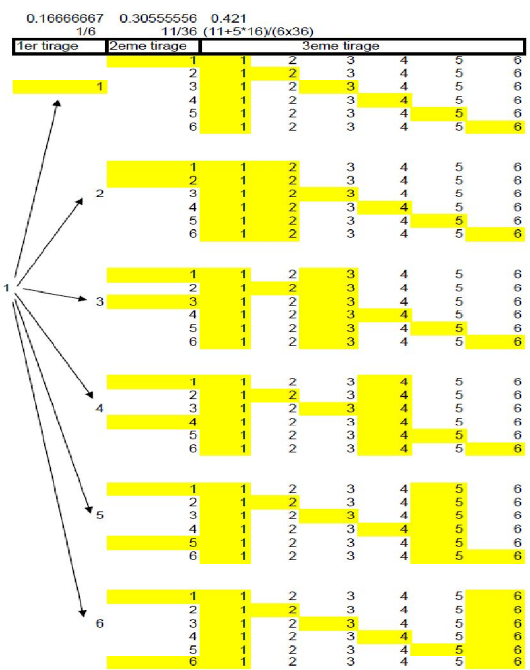


Figure 9: Visualization of the Number of Possible Cases and of Total Number of Cases for the Third Rolls if we Suppose that the Numeral 1 is drawn at the First draw

The demonstration stays the same from any other number for the beginning of the game.

V-4 Example of Determination of the Probability P_i in the case of Sequences of Decimals as π, φ or e

Now, we study the probability P_i for infinity sequences of decimals of the transcendent number π, φ or e.

To obtain the probability curve, the steps are the following:

Step 1: The sequence of decimals is cut in k sections that correspond at the apparition of all the possible numerals at minimum one time. A section is stop when the last numeral not already drawn appears in the list of decimals studied.

Step 2: On each section j, at the rank i, we identify if the numerals considered is already drawn in the previous decimals or not. We write P_{ji} this number.

P_{ji}=1 =>yes the considered decimals is already drawn in the previous rank of decimals of this section j;
P_{ji} =0 =>no the considered decimals is not already drawn in the previous rank of decimals of this section j;

Step 3: We determine the probability of drawn again a decimal at the rank i $P_i$by a mean value on the group of k sections studied of the P_{ji} (see formula 16)

$$P_i = \frac{\sum_{j=1}^{k} P_{ji}}{k} \quad (16)$$

V-4.1 Study of Series of Decimals of the Number « π =3.14… »

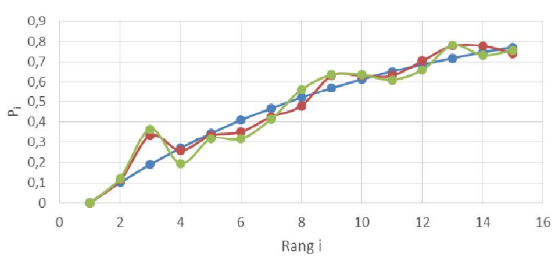
The table 10 and the figure 10 give the results obtained by the use of the expression (16) on 1281 and 1681 decimals of the numberπin view to determine the evolution of the probabilities.

In this case we have for the parameters:

n = 1; N = 10; g = 1.111

		Pi	
		Proba Mesurée	Proba Mesurée
Tirage Ti	Th evolution	1681 décimales	1281 décimales
1	0	0	0
2	0,1	0,111111111	0,12195122
3	0,19	0,333333333	0,365853659
4	0,271	0,259259259	0,195121951
5	0,3439	0,333333333	0,317073171
6	0,40951	0,351851852	0,317073171
7	0,468559	0,425925926	0,414634146
8	0,5217031	0,481481481	0,56097561
9	0,56953279	0,62962963	0,634146341
10	0,612579511	0,62962963	0,634146341
11	0,65132156	0,62962963	0,609756098
12	0,6861894	0,703703704	0,658536585
13	0,71757046	0,777777778	0,780487805
14	0,74581342	0,777777778	0,731707317
15	0,771232075	0,740740741	0,756097561

Table 10: Measured and Theoretical Values of P_i for the Numberπ at 1681 and 1281 Decimals



P_i Theoretical $P_i$for 1681 decimals $P_i$for 1281 decimals

Figure 10: Measured and Theoretical Probabilities $P_i$of old Decimals at the Rank i for the Number π at 1281 and 1681 Decimals

V-4.2 Study of the Decimals of the Gold Number « φ=1.618… »

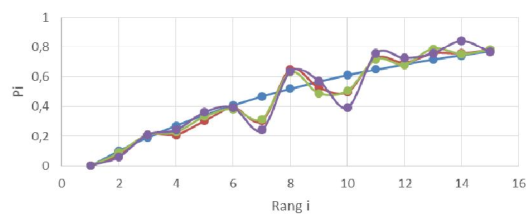
The table 11 and the figure 11 give the results obtained by the use of the expression (16) on 1000, 1755 and 2032 decimals of the gold numberφ in view to determine the evolution of the probabilities.

In this case we have for the parameters:

n = 1 ; N = 10 ; g = 1.111

		Probabilité Mesurée	Probabilité Mesurée	Probabilité Mesurée
Tirage Ti	Th evolution	2032 décimales	1755 décimales	1000 décimales
1	0	0	0	0
2	0,1	0,075757576	0,087719298	0,06060606
3	0,19	0,212121212	0,210526316	0,21212121
4	0,271	0,212121212	0,228070175	0,24242424
5	0,3439	0,303030303	0,333333333	0,36363636
6	0,40951	0,393939394	0,385964912	0,39393939
7	0,468559	0,303030303	0,315789474	0,24242424
8	0,5217031	0,651515152	0,631578947	0,63636364
9	0,56953279	0,53030303	0,49122807	0,57575758
10	0,61257951	0,5	0,50877193	0,39393939
11	0,65132156	0,727272727	0,719298246	0,75757576
12	0,6861894	0,696969697	0,684210526	0,72727273
13	0,71757046	0,757575758	0,789473684	0,75757576
14	0,74581342	0,757575758	0,754385965	0,84375
15	0,77123208	0,78125	0,781818182	0,76666667

Table 11: Theoretical and Measured P_i for the numberφ at 1000, 1755 and 2032 Decimals



P_i Theoretical $P_i$for 2032 decimals $P_i$for 1755 decimals Pi for 1000 decimals

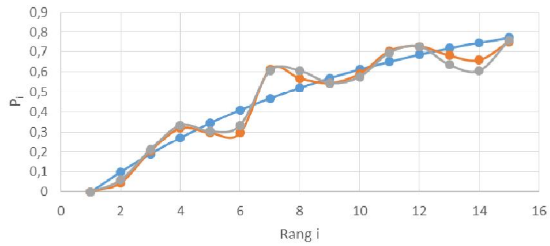
Figure 11: Measured and Theoretical Probabilities P_i of old Decimals at the Rank i for the gold Number φ at 1000, 1755 and 2032 Decimals

V-4.3 Study of the Decimals of the Euler Number « e=2.718…»

The table 12 and the figure 12 give the results obtained by the use of the expression (16) on 1019 and 1350 decimals of the numberein view to determine the evolution of the probabilities. In this case we have for the parameters: n = 1; N = 10; g = 1.111

Nombre e		Pi Proba Mesurée	Pi Proba Mesurée
Tirage Ti	Th évolution	1350 décimales	1019 décimales
1	0	0	0
2	0,1	0,045454545	0,060606061
3	0,19	0,204545455	0,212121212
4	0,271	0,318181818	0,333333333
5	0,3439	0,295454545	0,303030303
6	0,40951	0,295454545	0,333333333
7	0,468559	0,613636364	0,606060606
8	0,5217031	0,568181818	0,606060606
9	0,56953279	0,545454545	0,545454545
10	0,61257951	0,590909091	0,575757576
11	0,65132156	0,704545455	0,696969697
12	0,6861894	0,727272727	0,727272727
13	0,71757046	0,681818182	0,636363636
14	0,74581342	0,659090909	0,606060606
15	0,7712308	0,75	0,757575758

Table 12: Theoretical and Measured P_i for the Number e at1350 and 1019 Decimals



P_i Theoretical P_i for 1350 decimals P_i for 1019 decimals

Figure 12: Measured and Theoretical Probabilities P_i of old Decimals at the Rank i for the Number e at 1350 and 1019 Decimals

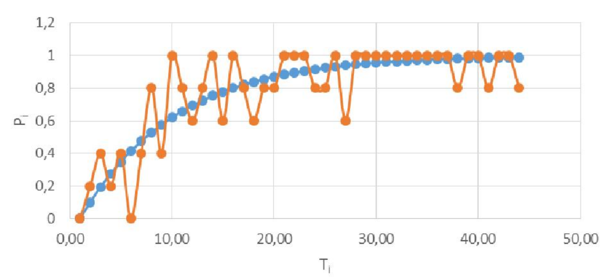
We observe an excellent correlation between the theoretical values issued of the evolution law (formula 15) and the measured value of P_i obtain after analysis of more than 1000 successive random decimals (formula 16). We see also that the shape of the curve is rather stable in function of the number of decimals studies and also in function of the transcendant number study.

V-5 Example of Determination of the Probability P_i for a Seriesof drawn of the Game of Chance Loto

The table 13 and the figure 13 give the results obtained by the use of the expression (14) on 44 successive draws of the game of chance Loto. In this case we have for the parameters: n= 5; N = 49; g = 1.1136363636

N° de Tirage	Pi théorique	Pi mesuré	N° de Tirage	Pi théorique	Pi mesuré
1	0	0	23	0,90632038	1
2	0,10204082	0,2	24	0,91587952	0,8
3	0,1936693	0,4	25	0,92446325	0,8
4	0,27594795	0,2	26	0,93217108	1
5	0,34983081	0,4	27	0,9390924	0,6
6	0,4161746	0	28	0,94530746	1
7	0,47574862	0,4	29	0,95088833	1
8	0,52924366	0,8	30	0,95589973	1
9	0,57728002	0,4	31	0,96039975	1
10	0,62041472	1	32	0,96444059	1
11	0,65914791	0,8	33	0,96806911	1
12	0,69392873	0,6	34	0,97132736	1
13	0,7251605	0,8	35	0,97425314	1
14	0,75320534	1	36	0,97688037	1
15	0,77838847	0,6	37	0,97923952	1
16	0,80100189	1	38	0,98135793	0,8
17	0,82130782	0,8	39	0,98326018	1
18	0,83954172	0,6	40	0,98496833	1
19	0,85591501	0,8	41	0,98650217	0,8
20	0,87061756	0,8	42	0,9878795	1
21	0,88381985	1	43	0,98911629	1
22	0,89567497	1	44	0,99022687	0,8

Table 13: Theoretical and Measured Values of the Probability / Proportion of old Numbersp$_i$ at each draw by Comparison with a Reference draw for a Game Type Loto



P_i Theoretical following Theory of the evolution P_i measured

Figure 13: Comparison of the Theoretical and Measured Value of the Probability of Drawn old Number on 44 Successive draws of the Loto

VI Propositions of Theorems

Based on the previous results, we can propose the 2 following theorems:

Theorem n°1

If we have a game with N different numerals. If the aim of the game is to draw n numbers at each draw,
If after each draw of n numbers, all the numbers are put again in the game,
If we consider a series of consecutive draws of this game computed from a reference draw taken arbitrary on this list of draws and corresponding strictly at the release of all the N possible numerals of the game,

So, the probability/proportion $P_i$of numerals already drawn at a draw T_i(said also old number) by comparison with the departure draw taken arbitrary take the following value:

$$P_i = 1 - \frac{(N-n)^{Ti-1}}{N^{Ti-1}} = 1 - \frac{N(N-n)^{Ti}}{N^{Ti}(N-n)} = \frac{N^{Ti} - N(N-n)^{Ti-1}}{N^{Ti}}$$

Demonstration

This theorem have been demonstrate on the dame heads and tails and on the game of dice at the paragraphs V.2 and V.3.

Theorem n°2

If we have an infinite sequence of decimals of a transcendant number as (π, φ,eetc) with N different numerals(10 , 0 to 9) cut in k section corresponding strictly at the release at minimum one time of the N possible numbers. If for each section j, we determine the P_{ij}, the identification number of decimals already drawn or not at the rank I of the section j (P_{ji}=1 => yes the considered decimals is already drawn in the previous rank of decimals of this section j and P_{ji} =0 => no the considered decimals is not already drawn in the previous rank of decimals of this section j) The P_{ij} are established on the k section considered of the decimals series. So, the probability/proportion P_i of numerals already drawn at a rankT_i by comparison with the reference on any section j takesin mean the following value on the group k of sections studied:

$$P_i = 1 - \frac{(N-n)^{Ti-1}}{N^{Ti-1}} = 1 - \frac{N(N-n)^{Ti}}{N^{Ti}(N-n)} = \frac{N^{Ti} - N(N-n)^{Ti-1}}{N^{Ti}} \approx \frac{\sum_{j=1}^{k} P_{ji}}{k}$$

Demonstration

This theorem have been demonstrated on the transcendant and irrational numberπ andφ and e respectively at the chaptersV.4.1, V.4.2, V.4.3 on more than 1000 decimals.

VII. Conclusions

We have shown that from the evolution law established by LNottale, J Chaline et P Grou, it is possible to define a law that predict the evolution in the time of the cumulative number of new numerals (see formula 13) from a referential draw. By consequences it's also possible to determine from this law the cumulative number of old numerals (already drawn in previous draw).This referential draw is taken arbitrary in a series of consecutive draws. The series of draws studied correspond at the time that all the possible numbers of the games (N) or of the decimals series appears all at minimum on time. From this law we have found the probability P_i of release of old numbers at each draw by comparison with a referential draw (see formula 14 at 16). This law is applicable at the decimal series of transcendant number asπ or φ or e that at random as (Lotoetc). Philosophically, the main and logical consequence of this analysis is that it seems that the numerals that appears at each draw in randomness decimal series of transcendant number or in of series draw of game of chance, are in fact dependent of the previous draws. It seems so that randomness must have a sort of memory of the previous numerals release in view to ensure a certain probability/proportion P_i of old numerals at each successive draw. This probability of old number at each draw seem predictable via the evolution law. (See formula 14 to 16). In the works carried out by the author of this article since more than 15 years, based on an analysis of several thousand of decimal of transcendent number, the previous results seems already verified. In its famous sentence « god does not play dice » Einstein would like to say that random have no place in the formulation of the mechanics. The quantic mechanic seems to prove the contrary. But after reading this study, if in fact Einstein have partially right…

References

R.CASH, J.CHALINE, L.NOTTALE, GROU P., « Développement humain et loi log-périodique », Comptes Rendus de l'Académie des Sciences, Biologies, 325, 2002, p. 585-590.

J.CHALINE, L.NOTTALE, GROU P., « L'arbre de vie a-t-il une structure fractale ? », Comptes Rendus de l'Académie des Sciences, Paléontologie, 328, 1999, p. 717-726.

J.-FGOUYET, Physique et structures fractales, Paris, Masson, 1996.

P.GROU, L.NOTTALE, J.CHALINE, Évolution morphologique et culturelle humaine ; l'apport des modèles fractals, Colloque, Aix en Provence, 2002, p. 1-10.

A.JOHANSEN, D.SORNETTE, “Finite-time singularity in the dynamics of the world population and economic indices”, Physica A, 294, 2001, p. 465-502.

P.GROU P, L.NOTTALE, J.CHALINE J, « Les arbres de l'évolution», Paris, Hachette Sciences, 2000, [Communications privées].

P.GROU, L.NOTTALE, J.CHALINE J, “On the fractal structure of evolutionary trees”, Fractals in Biology and Medecine, Proc. Third Int. Symposium, March 8-11, 2000, p. 247-254.

D.SORNETTE, “Discrete scale invariance and complex dimensions”, Physics Reports, 297, 1998, p. 239-270.

D.SORNETTE, Why Stock Markets Crash: critical events in complex financial systems, Princeton, Princeton University Press, 2003.

D.SORNETTE, A.A.JOHANSEN, J.P.BOUCHAUD, “Stock market crashes, precurses and replicas”, Journal of Physics I, France, 6, 1996, p.167-175.

D.SORNETTE, C.G.SAMMIS, “Complex critical exponents from renormalization group theory of earthquakes”, Journal of Physics I, France, 5, 1995, p. 607-619.

I.BRISSAUD, “ La chronologie du Jazz suit-elle une loi log-périodique?”, Marh.Sci.hum – Marhématics and Social Sciences ”, 45e année, n°178, 2007(2), p. 41-50.

J.CHALINE, L.NOTTALE, GROU P., « La loi qui prédit l’évolution » Eurekaseptembre 1999

M.FORRIEZ, P.MARTIN « De l’utilité de la théorie de la relativité d’échellede L. Nottale en géographie - Partie 2 : Application d’un modèle scalaire spatio-temporel »

Université d’Avignon - UMR ESPACE

J.CHALINE, L.NOTTALE, GROU «P. L’arbre de la vie a-t-il une structure fractale » Académie des sciences / Elsevier, ParisPaléontologie / Palaeontology