An Evaluation of Errors in Energy Forecasts by the SARFIMA Model

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Abstract

Forecasting is tricky business. This is particularly true in the energy field, where the highly random behavior of energy prices and technological change make forecasting difficult. In this paper, we study the long memory characteristic using Higuchi method and establish *SARFIMA model to forecast consumption of petroleum products in U.S. Furthermore, we apply an error decomposition technique to study errors in energy forecasts by the SARFIMA model. The results indicate the SARFIMA model give precise predictions.*

Key words: SARFIMA model, forecast error, energy forecast, Higuchi method

1. Introduction

At the beginning of the 20th century, petroleum was a minor resource used to manufacture lubricants and fuel for kerosene and oil lamps. One hundred years later it had become the preeminent energy source for the U.S. and the rest of the world. World wide consumption of petroleum was 85.4 million barrels per day in 2009. The three largest consumption countries were:

- United Sates (18.7 million barrels per day)
- China (8.12 million barrels per day)
- Japan (4.4 million barrels per day)

The United States is the largest energy consumer in the world. The United States consumes more energy from petroleum than from any other energy source. In 2009 total U.S. petroleum consumption was 18.7 million barrels per day and the United States consumed a total of 6.9 billion barrels of oil (refined petroleum products and bio fuels), which was about 27% of total world oil. Energy consumption has increased at a faster rate than energy production over the last fifty years in the U.S. (when they were roughly equal). This difference is now largely met through imports (IEO 2010). Sakhabakhsh and Yarmohammadi (2012) indicated SARFIMA model is an appropriate model for energy modeling. In this paper, we evaluate forecasts errors of SARFIMA model in the energy field. As an illustration, we consider monthly consumption of petroleum products of U.S.

The presence of long memory in the time series provides a researcher the needed information to transform a non-stationary series into a stationary one. Therefore it is critical to explore the presence or otherwise of long memory in consumption of petroleum products of U.S. This is done through the application of Higuchi method (Higuchi, 1988). Several estimation methods are available. However, the Higuchi approach clearly outperforms the other methods. This is probably due to the fact that the Higuchi method evaluates the cumulative sum of the data to convert the series from a noise to a motion.

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Then we conduct our analysis using SARFIMA model's energy forecasts with time horizons ranging from 1 to 15 years made during the period 1994 through 2009.

The recent finance and economic literature has recognized the importance of long memory in analyzing time series data. A long memory can be characterized by its autocorrelation function that decays at a hyperbolic rate. Such a decay rate is much slower than that of the time series, which has short memory. Traditional Box-Jenkins models describing short memory, such as AR (p), MA (q), ARMA (p, q), and ARIMA (p, d, q) can not describe long memory precisely. A set of models has been established to overcome this difficulty, and the most famous one is the autoregressive fractionally integrated moving average (ARFIMA or ARFIMA (p, d, q)) model. ARFIMA model was established by Granjer and Joyeux (1980). An overall review about long memory and ARFIMA model was model by Baillie (1996). In many practical applications researchers have found time series exhibiting both long memory and cyclical behavior. For instance, this phenomenon occurs in revenues series, inflation rates, monetary aggregates, and gross national product series.

Consequently, several statistical methodologies have proposed to model this type of data including the Gegenbauer autoregressive moving average processes (GARMA), k-factor GARMA processes, and seasonal autoregressive fractionally integrated moving average (SARFIMA) models. The GARMA model was first suggested by Hosking (1981) and later studied by Gray et al (1989) and Chung (1996). Other extension of the GARMA process is the k-factor GARMA models proposed by Giratis and Leipus (1995) and Woodward et al (1998). This paper investigates a special case of the k-factor GARMA model, which is considered by Porter – Hudak (1990) and naturally extends the seasonally integrated autoregressive moving average (SARIMA) model of Box and Jenkins (1976). Katayama (2007) examined the asymptotic properties of the estimators and test statistics in SARFIMA models. There are several methods for estimating the parameters in time series models. In this paper, we estimate the parameters using conditional sum of squares (CSS) method.

2. Materials & Methods

2.1 Long memory model

The basic properties of processes with long memory are a hyperbolically decaying autocorrelation function (ACF), a spectral density increasing without limit as the frequency tends to zero, and the so-called Hurst phenomenon. The last characterization implies that the Hurst exponent (H), the parameter representing the probability that an event in a time series is followed by a similar event, deviates from 0.5. For $H = 0.5$, the observations are independent. There are two classes of fractal processes, which can be persistent or antipersistent: fractional Brownian motion (fBm) and fractional Gaussian noise (fGn). Mandelbrot and van Ness (1968) introduced fBm as a generalization of ordinary Brownian motion, a continuous-time stochastic process with independent increments. Brownian motion with H= 0.5 separates antipersistent and persistent fBms. H can be any real number in the range [0, 0.5) for antipersistent and (0.5, 1] for persistent series, where antipersistence implies negative correlations between the successive increments of a fBm series. fGn, a discrete-time analogue of fBm, was defined by Mandelbrot and Wallis (1969).

Gaussian noise is a stationary process with constant mean and variance, whereas Brownian motion is nonstationary with stationary increments. Differencing fBm creates fGn, and summing fGn produces fBm; the related processes are characterized by the same Hurst exponent. There are three major ways for capturing long memory: (1) by means of H within the scope of fractal analysis; (2) through the power exponent β of the power spectrum function $1/f^{\beta}$ in the spectral analysis; and (3) through the fractional-differencing parameter d within the ARFIMA framework. Figure 1 illustrates these concepts. Table 1 summarizes the relationships between β, H, and d.

Table 1: Relationships between parameters capturing long memory characteristic

	β	H	d
Random walk (ordinary Brownian motion)	2	0.5	1
Whitw noise (ordinary Gaussian noise)	0	0.5	0
fGn	[-1, 1] [-1, 0) antipersistent (0, 1] persistent	[0, 1] [0, 0.5) antipersistent (0.5, 1] persistent	[-0.5, 0.5] [-0.5, 0) antipersistent (0, 0.5] persistent
fBm	[1, 3] [1, 2) antipersistent (2, 3] persistent	[0, 1] [0, 0.5) antipersistent (0.5, 1] persistent	[0.5 , 1.5] [0.5, 1) antipersistent (1, 1.5] persistent

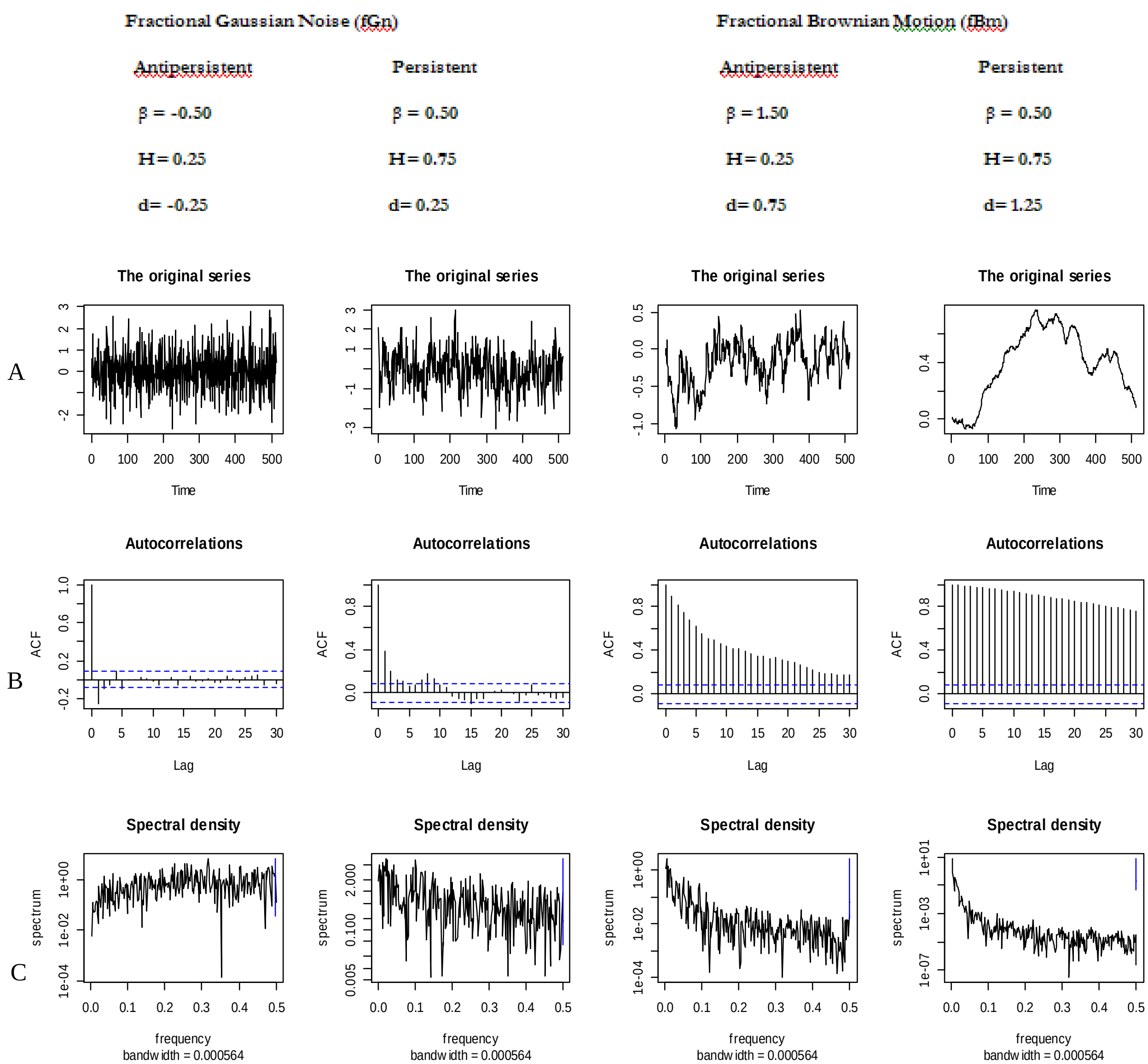

Fig. 1: (A) The original series (T = 512), (B) autocorrelation functions (ACFs), and (C) spectral density plots.

2.1.1 ARFIMA (p, d, q) process

In this section, we present the ARFIMA (p, d, q) model (also called Fractional ARIMA model) and some related theoretical results. Models that include fractional differentiation d in the interval (0, 0.5) are able to represent any time series that shows persistence. Initial studies of time series with long memory characteristics were given by Hurst (1951).

ARFIMA processes are a generalization of the ARMA and ARIMA models. Persistence or long memory property has been observed in time series from different fields such as metrology, astronomy, hydrology, and economy.

One can characterize the persistence by two different forms:

• In time domain, the autocorrelation function $\rho_X(.)$ decays hyperbolically to zero, that is:

$$\rho_X(k) \cong k^{2d-1}, \text{ when } k \to \infty. \quad (1)$$

• In frequency domain, the spectral density function $f_X(.)$ is unbounded when the frequency is near zero, that is:

$$f_X(\omega) \cong \omega^{-2d} \text{ when } \omega \to 0. \quad (2)$$

One of the models that can describe the persistence is the so-called ARFIMA (p, d, q) processes.

Definition 1: A stochastic process $\{X_t\}_{t\in\mathbb{Z}}$ is Gaussian if, for any set of $t_1, t_2, \dots, t_n \in \mathbb{Z}$, the random variables $X_{t_1}, X_{t_2}, \dots, X_{t_n}$ have a n-dimensional normal distribution.

We observe that weakly stationary process $\{X_t\}_{t\in\mathbb{Z}}$ dose not need to be strongly stationary. However, any weakly stationary Gaussian process will be also strongly stationary.

Definition 2: The process $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is said to be a white noise process with zero mean and variance σ_ε^2, denoted by $\varepsilon_t \sim wn(0, \sigma_\varepsilon^2)$, if

$$E(\varepsilon_t) = 0, ar(\varepsilon_t) = E(\varepsilon_t^2) = \sigma_\varepsilon^2, \text{ and} \quad (3)$$

$$\gamma_\varepsilon(k) = \begin{cases} \sigma_\varepsilon^2, & k = 0, \\ 0, & k \neq 0. \end{cases}$$

Definition 3: Let $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ be a white noise process with zero mean and variance $\sigma_\varepsilon^2 > 0$, and B the backward-shift operator, i.e., $B^k(X_t) = X_{t-k}$. If $\{X_t\}_{t\in\mathbb{Z}}$ is a linear process satisfying:

$$\Phi(B)(1-\mathrm{B})^d x_t = \Theta(B)\varepsilon_t, \quad t \in \mathbb{Z} \quad (4)$$

Where d ∈ (-0.5,0.5), $\Phi(.)$, $\Theta(.)$ are polynomials of degree p and q, respectively, given by:

$$\Phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p, \quad (5)$$
$$\Theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q$$

Where $\phi_i, 1 \leq i \leq p,\ \theta_j, 1 \leq j \leq q,$ are real constants, than $\{X_t\}_{t\in\mathbb{Z}}$ is called general fractional differentiation ARFIMA (p, d, q) process, where d is the degree or fractional differentiation parameter.

The term $(1-B)^d$, for d$\in \mathbb{R}$, is defined through the binomial expansion:

$$(1-\mathrm{B})^d = \sum_{k=0}^{\infty}\binom{d}{k}(-B)^k = 1 - dB - \frac{d}{2!}(1-d)B^2 \dots . \qquad (6)$$

If d ∈ (-0.5, 0.5), then $\{X_t\}_{t\in\mathbb{Z}}$ is a stationary, and an invertible process. The most important characteristic of an ARFIMA (p,d,q) process is the property of long dependence, when d ∈ (0, 0.5), short dependence, when d=0, and intermediate dependence, when d ∈ (-0.5, 0).

2.1.2 SARFIMA (p, d, q) (P, D, Q)$_s$ processes

In many practical situation time series exhibit a periodic pattern. We shall consider the SARFIMA (p, d, q) (P, D, Q)$_s$ process, which is an extension of the ARFIMA process.

Definition 1: Let $\{X_t\}_{t\in\mathbb{Z}}$ be a stationary stochastic process with spectral density function $f_x(.)$.suppose there exists a real number $b \in (0,1)$, a constant $C_f > 0$ and one frequency $G \in [0,\pi]$ (or a finite number of frequencies) such that:

$$f_x(\omega) \sim c_f |\omega - G|^{-b}, \text{ when } \omega \to G. \qquad (7)$$

Then, $\{X_t\}_{t\in\mathbb{Z}}$ is a long memory process.

Remark 1: In Definition 1, when$b \in (-1,0)$, we say that the process $\{X_t\}_{t\in\mathbb{Z}}$ has the intermediate dependence property (Doukhan et al., 2003).

Definition 2: Let $\{X_t\}_{t\in\mathbb{Z}}$ be a stochastic process given by the equation:

$$\phi(B)\Phi(B^s)(1-B)^d(1-\mathrm{B}^s)^D(x_t - \mu) = \theta(B)\Theta(B^s)\varepsilon_t, \quad t \in \mathbb{Z} \qquad (8)$$

Where $\mu$the mean of the process is, $\{\varepsilon_t\}_{t\in\mathbb{Z}}$ is a white noise process with zero mean and variance $\sigma_\varepsilon^2 = \mathbb{E}(\varepsilon_t^2), s \in \mathbb{N}$ is the seasonal period; B is the backward-shift operator, that is:

$$B^{sk}(\mathrm{X}_t) = X_{t-sk} \qquad (9)$$

The seasonal difference operator ∇_S^D is defined as:

$$\nabla_S^D = (1-B^S)^D \qquad (10)$$

$\phi(.)$, $\theta(.)$, $\Phi(.)$, and $\Theta(.)$ are the polynomials of degrees p, q, P, and Q, respectively, defined by:

$$\phi(B) = \sum_{i=0}^{p} (-\phi_i) B^i \quad (11)$$

$$\theta(B) = \sum_{j=0}^{q} (-\theta_j) B^j \quad (12)$$

$$\Phi(B) = \sum_{k=0}^{P} (-\Phi_k) B^k \quad (13)$$

$$\Theta(B) = \sum_{l=0}^{Q} (-\Theta_l) B^l \quad (14)$$

Where, $\phi_i, 1 \le i \le p, \theta_j, 1 \le j \le q, \Phi_k, 1 \le k \le P$, and $\Theta_l, 1 \le l \le Q$ are constants and $\phi_0 = \Phi_0 = -1 = \theta_0 = \Theta_0$.

Then, $\{X_t\}_{t\in\mathbb{Z}}$ is a seasonal fractionally integrated ARMA process with period s, denoted by SARFIMA (p, d, q) $(P, D, Q)_s$, where d and D are the order of differencing and the seasonal differencing respectively.

Theorem 1: Let $\{X_t\}_{t\in\mathbb{Z}}$ be a SARFIMA (p, d, q) $(P, D, Q)_s$ process given by the Eq. (8), with zero mean and seasonal period $s \in \mathbb{N}$.

Suppose $\phi(z)\Phi(z^s) = 0$ and $\theta(z)\Theta(z^s) = 0$ have no common zeroes. Then, the following are true.

(i) The process $\{X_t\}_{t\in\mathbb{Z}}$ is stationary if d + D < 0.5, D < 0.5 and $\phi(z)\Phi(z^s) \neq 0$, for $|z| \le 1$.

(ii) The stationary process $\{X_t\}_{t\in\mathbb{Z}}$ has a long memory property if 0 < d + D < 0.5, 0 < D < 0.5 and $\phi(z)\Phi(z^s) \neq 0$, for $|z| \le 1$.

(iii) The stationary process $\{X_t\}_{t\in\mathbb{Z}}$ has an intermediate memory property if -0.5 < d + D < 0, -0.5 < D < 0 and $\phi(z)\Phi(z^s) \neq 0$, for $|z| \le 1$ (Bisognin and Lopes, 2009).

2.2 Higuchi method

In order to determine the presence of long memory in consumption of petroleum products of U.S, the Higuchi method is applied. In Higuchi method, the series is assumed to have the character of a noise, not a motion. The series is partitioned into m groups. The cumulative sums of the series are evaluated to convert the series from a noise to a motion. Absolute differences of the cumulative sums between groups are analyzed to estimate the fractal dimension of the path. The number of groups, m, is increased and the process is repeated. The result changes with increasing m in a way related by Higuchi's theory to the Hurst parameter H of the input series.

A log-log plot of the statistic versus number of groups is, ideally, linear, with a slope related to H, so H can be determined by linear regression. The Hurst exponent, H, helps to infer the presence or otherwise of long memory in a time series. If $0.5 < H \le 1.0$, then the time series is said to be persistent. Persistent time series are also referred to as long memory characteristic.

2.3 Conditional Sum of Squares method

There are several methods for estimating the parameters in time series models. In this paper, we implement the CSS method to estimate the SARIMA and SARFIMA models of consumption of petroleum products of U.S. This method is equivalent to the full Maximum Likelihood Estimator (MLE) under quite general conditional homoskedastic distributions. A description of the properties of the CSS estimator and its finite sample performance is presented in Chung and Baillie (1993).

2.4 Evaluation of Forecast Errors

In this paper, we apply an error decomposition technique to study errors in energy forecasts by the SARFIMA model. We are interested in understanding these errors as related to various forecasts time horizons years. We use two metrics to determine forecast error: mean percentage error and mean absolute percentage error. Mean percentage error (MPE) is an average error of all forecasts of a given forecast horizon and is given by the function:

$$MPE_{\tau} = \frac{\sum \frac{(\hat{y} - y)}{y}}{n_{\tau}} \tag{15}$$

Where τ is our forecast horizon (1year, 2years, …, 15 years); $\hat{y}$ is our forecasted value for period τ; y is our actual value for period τ; and n_{τ} is the number of forecasts with time horizon τ. MPE calculations for a single forecast horizon (τ) could take on a positive or negative value. If MPE > 0, then the forecast value was higher than the actual value, and the forecast represents an overestimate. If MPE < 0, then the forecast value was less than the actual value, and the forecast is an underestimate. The reader should note that an average MPE near zero does not imply a near perfect forecast. The average may be close to zero, but may represent a combination of highly overestimated and underestimated forecasts that cancel each other out on average. To more clearly explore the accuracy of forecasts, without concern over whether forecasts are underestimated or overestimated, we apply the mean absolute percentage error (MAPE), given by the following function:

$$MAPE_{\tau} = \frac{\sum \left| \frac{(\hat{y} - y)}{y} \right|}{n_{\tau}} \tag{16}$$

Where the variables and indices remain the same as in Eq. 15. Both MPE and MAPE identify forecast errors.

3. Empirical Study

3.1 The data

In this study, we will use the monthly consumption of petroleum products of U.S during (Jan 1994 to Dec 2009). The data are obtained from the energy information administration of the U.S. department of energy. Figure 2 displays the data of consumption of petroleum products of U.S., $\{x_t\}$.

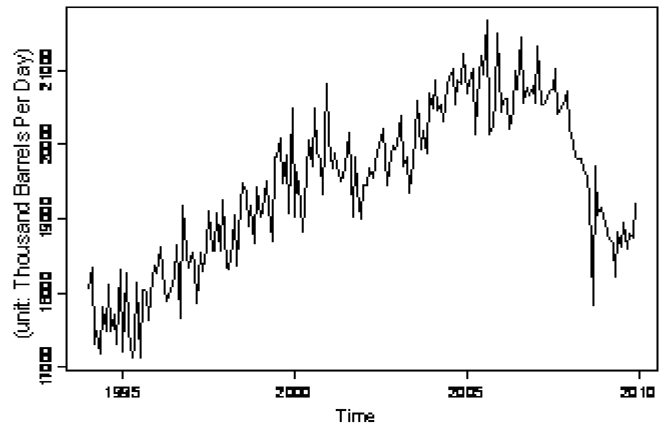

Fig. 2: Time plot of consumption of petroleum products in U.S.

3.2 Test of long memory

We used Higuchi analysis, and obtained the Hurst exponent H = 0.9507, as shown in Figure 3, which indicated strong long memory in the consumption of petroleum products of U.S. Test of Higuchi is obtained by fractal package in R (R Development Core Team), a popular and freely available software package frequently used in the applied social and behavioral sciences.

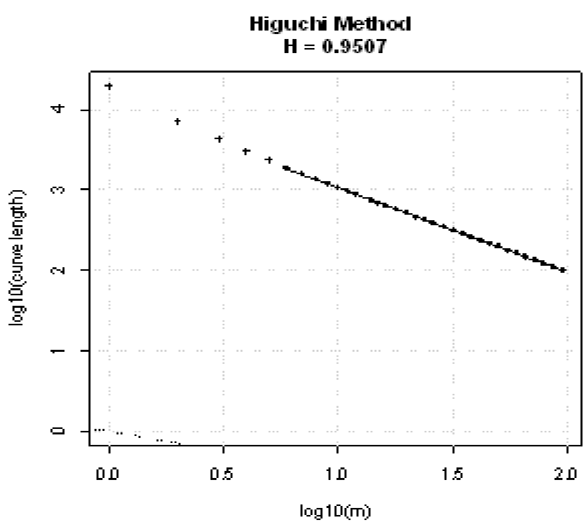

Fig. 3: Higuchi method result

3.3 Parameter estimation and establishment of SARFIMA and SARIMA models

Consumption of petroleum products of U.S. exhibit a periodic pattern and contain long memory characteristic therefore SARFIMA model is fitted into the data. If time series exhibit long memory property, forecasts values based on SARIMA model may not be reliable. Therefore to search for the best representation of this data, we compared SARFIMA and SARIMA models in terms of AIC. We first fitted data by the CSS method, where we used a sample mean of $\{y_t\}$, $\bar{y}$ as an estimator of $E[y_t] = \mu$, and set s = 12. AIC criteria are also used under the assumption of normality (Brockwell and Davis, 1991). All calculations were made using S-PLUS.

Table 2 shows the SARFIMA and SARIMA models in terms of AIC model selection with estimators. The numbers in parentheses in the column of AIC denote the ranking of models in terms of AIC.

Table 2: Summary of AIC model selection and estimates

AIC	d	D	ϕ_1	ϕ_2	θ_1	θ_2	Φ_1	Θ_1
(1)2634.2	1	0.306	0	0	0.7	-0.1	0	0
(2)2799.4	1	0	-0.6	-0.4	0	0	0.9	-0.7

From the Table 2 the SARFIMA (0, 1, 2) (0, 0.306, 0)$_{12}$ model is the appropriate model in terms of AIC among the 2 models candidates. Since the number of observations was 192, we used 168 of them in building SARFIMA and SARIMA models, and the rest were used to compare with the forecasting results. We made a 24-steps ahead forecast with SARFIMA (0, 1, 2)(0, 0.306, 0)$_{12}$ and SARIMA (2, 1, 0)(1, 0, 1)$_{12}$, and compared the forecasting values with the real observations. The results are shown in Figure 4.

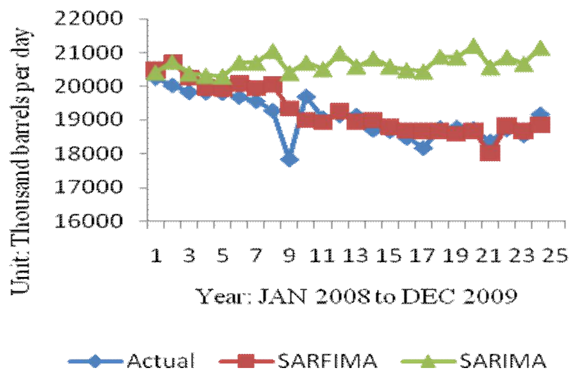

Fig. 4: Comparison of forecast performance of SARFIMA and SARIMA models

As is shown in the Figure 5, the estimated forecast values from SARFIMA model are closest to real data than SARIMA model. Therefore, we can conclude that the forecast performance of SARFIMA model is better than SARIMA model.

3.4 Forecasting

The appropriate model is SARFIMA (0, 1, 2) (0, 0.306, 0)$_{12}$ model which is used to predict the consumption of petroleum products of U.S. till the end of 2012 and 2020 as shown in Figure 5 and Figure 6. The results of in-sample forecasts of the SARFIMA model are shown in Table 3.

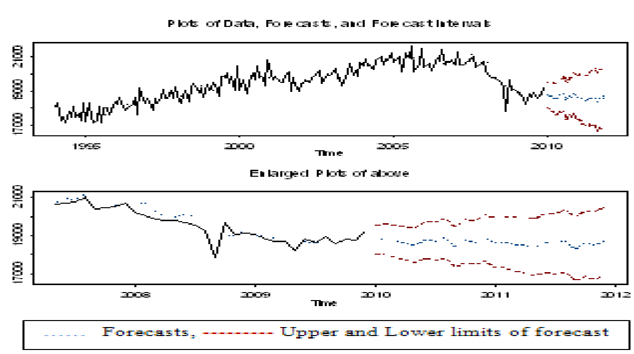

Fig. 5: Prediction plot of consumption rate of petroleum products of U.S. (2010-2012)

Table 3: In-sample forecasts for the SARFIMA (0, 1, 2) (0, 0.306, 0)$_{12}$ model

Date	Actual	Forecasts	Error
2009-01	19040	18939.76	100.24
2009-02	18822	18999.25	-177.25
2009-03	18719	18792.96	-73.96
2009-04	18672	18678.92	-6.92
2009-05	18211	18676.08	-465.08
2009-06	18828	18663.34	164.66
2009-07	18626	18588.25	37.75
2009-08	18949	18669.82	279.18
2009-09	18594	18023.80	570.2
2009-10	18803	18828.88	-25.88
2009-11	18753	18672.28	80.72
2009-12	19237	18859.32	377.68

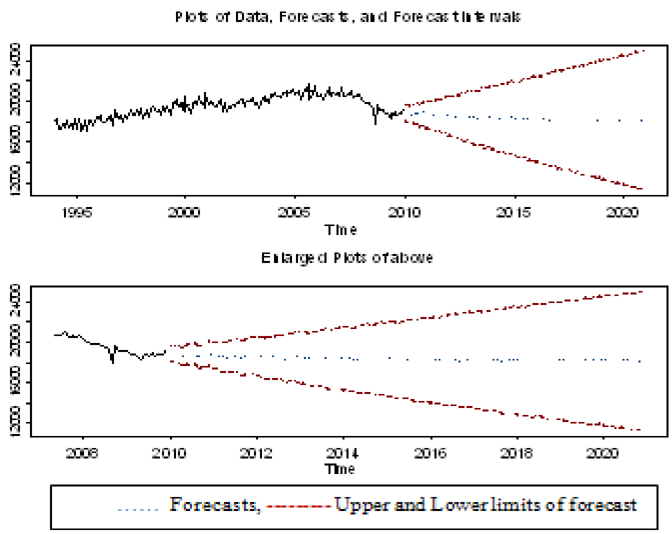

Fig. 6: Prediction plot of consumption rate of petroleum products of U.S. (2010-2020)

3.5 Analysis of MAPE

This analysis offers a closer look at the general accuracy of forecasts, by the SARFIMA model, for time horizons ranging from 1 to 15 years, we can use this analysis to determine if forecasts exhibit increased uncertainty when time horizons are lengthened. Table 4 presents both MAPE and MPE calculations. The results from the MAPE analysis are shown in Figure 7 demonstrate that forecasts have relatively small errors for the time horizons analyzed (ranging from 1 to 15 years).

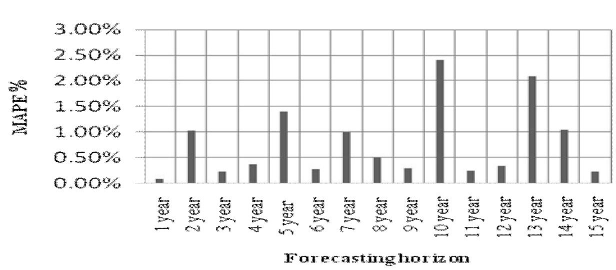

Fig. 7: MAPE for consumption of petroleum products by forecast length

Table 4: Prediction errors of consumption of petroleum products by the SARFIMA

	Forecast horizon (years)														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of observation	16	8	6	4	4	3	3	2	2	2	2	2	2	2	2
MPE	-0.4%	0.3%	0.06%	0.002%	-1.1%	0.2%	1%	0.1%	0.08%	-2.1%	0.25%	0.34%	-1.7%	1.05%	0.2%
MAPE	0.09%	1.03%	0.23%	0.37%	1.4%	0.2%	1%	0.5%	0.3%	2.4%	0.25%	0.34%	2.1%	1.05%	0.2%

3.6 Analysis of MPE

The MPE analysis expands on the MAPE analysis by identifying the directionality of forecast error. This analysis might point to a systemic problem with the forecast models used for a given time horizon. MPE calculations are shown in Table 5 and Figure 8. For forecasts between 1 to 15 years in length, the errors are small on average (around 2%). For forecasts of 1 year, 5 years, 10 years and 13 years, errors are negative (representing underestimation). For another time horizons, errors are positive (representing overestimation).

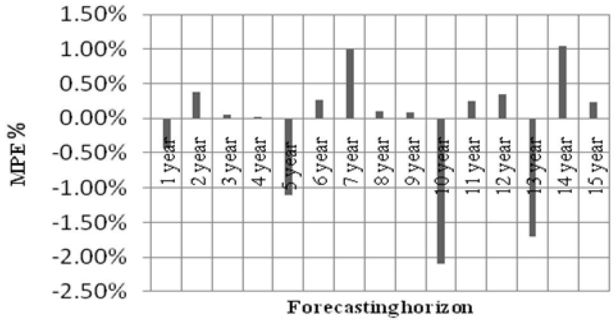

Fig. 8: MPE for consumption of petroleum products by forecast length

4. Conclusions

This paper has examined a seasonal long memory process, denoted as the SARFIMA model. We evaluated forecast errors of SARFIMA model in the energy field. As an illustration, we considered monthly consumption of petroleum products of U.S. The results indicated the appropriate model was SARFIMA (0, 1, 2) (0, 0.306, 0)$_{12}$ which was used to predict the data. Also both MPE and MAPE calculations identified the errors are small. On the basis, we concluded that the SARFIMA model is effective and give precise predictions.

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