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Interpolation of the Option Portfolio Valuation

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Abstract

In the option portfolio management, when the portfolio is complex, sometimes it is too expensive to revalue the whole portfolio computationally. One common practice is to interpolate the portfolio valuation using the interpolation based on a few pivot points. In this research, we are going to compare four interpolation approaches: Polynomial Fit, Cubic Spline, Black Scholes Fit, and Black Scholes Inverse. The Black Scholes Fit, Black Scholes Inverse are new approaches. We will demonstrate the conclusion with simulation results on some sample portfolios.

Keywords:

1. Introduction

Large hedge funds and investment banks have complex derivative portfolios. Sometimes a full revaluation could cost undesirable amount of computational power. Also, sometimes for risk management purposes, or for portfolio construction decision making purposes, it is necessary to experiment with new ideas and new concepts before the full implementation of the model.In these situations, interpolation of the portfolio value is an useful tool.

The basic idea of the interpolation of the portfolio value is to perform a full revaluation of the portfolio at a few pivot market spot levels. For simplicity, let us measure the market level with the SP500 index. Let P(x) function be the portfolio value function at x, assuming a full revaluation of the portfolio. Let Q(x) function be the interpolation function at x. Let S = {20%, 15%, 10%, 5%, 0%, -5%, -10%, -15%, -20%} be the 9 pivot points. The calibration process in the interpolation approaches will match / fit {(x, Q(x)) | x in S} to {(x, P(x)) | x in S}, assuming a full revaluation of P(x) at those 9 pivot points. The graph / function P(x) is often referred as the spot profile. In this research, the spot profile would be used to estimate delta, gamma of the portfolio. As another application of the spot profile, it would be used to derive the impact on the portfolio if certain hedging action is taken.

We will compare following four interpolation methods, Polynomial Fit, Cubic Spline, Black Scholes Fit, and Black Scholes Inverse. In the Polynomial Fit method, the Q function is a polynomial function passing through all 9 pivot points. In the Cubic Spline method, the Q function is a section wise cubic function passing through all 9 pivot points and smoothly connected between the sections. The Black Scholes Fit and the Black Scholes Inverse are two methods introduced here for the first time in the literature. The Black Scholes model, which was developed by Black and Scholes in 70's [1], is fundamental in modern financial theory. Later, many authors derived and formulated the Black-Scholes formula [2], [3], [4], [5]. In the Black Scholes Fit, a fitting portfolio of 9 "long" call options are chosen. A goal seek process tries to match the fitting portfolio to the original portfolio at all 9 pivot points. In the Black Scholes Inverse approach, the fitting portfolio is permitted to have both long and short options. This will allow to match the valuation of the fitting portfolio with the original portfolio exactly through a 9x9 matrix inverse. The interpolation is the valuation with the fitting portfolio.

In the two Black Scholes approaches, we will utilize some basic statistics of the portfolio on maturity and strike. We will investigate the effectiveness of the two new approaches and compare them with the Polynomial Fit and the Cubic Spline interpolation.

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2. The Model

2.1 Sample Portfolio Construction

For simplicity, we will construct a sample portfolio with only call options, with up to 250 positions of various maturities and moneyness. We assume the maturities are within a band, with a minimum maturity and a maximum maturity. As a baseline, a minimum maturity would be set at a quarter year and a maximum maturity at2.5 years. For simplicity, the options are always out of money. The moneyness of the option is within a band, with a minimum moneyness and a maximum moneyness. As a baseline, minimum moneyness would be set at 100% of the spot and the maximum moneyness at 140% of the spot. Note that we will study the sensitivity to bands of the maturity and the moneyness. We will study the effectiveness of interpolation methods to the size of the portfolio as well.

2.2 Interpolation Methodologies

The Cubic Spline methodology and the Polynomial Fit methodology are well known. In the case of Cubic Spline, Q(x) is a section wise cubic polynomial. In Polynomial Fit methodology, Q(x) is a8-degree polynomial.

We will introduce two new approaches using the Black-Scholes formula. We are going to find nine options at the various strikes and maturities to match P(S).In the Black Sholes-Fit approach, we will restrict the portfolio to only having "long" options. In the Black Sholes-Inverse approach, both "long" and "short" options are permitted. We are going to calibrate the size of the options at those maturities and strikes, so that the fitting portfolio has the total market value matching the original portfolio for market levels in S. Q(x), the interpolated value, will be the full revaluation of the fitting portfolio at x.

Let T = {20%, 19%, 18% …. 1%, 0%, -1% …., -18%, -19%, -20%} be all the shock with 1% step from 20% to -20%. We will use the following measurement to compare the effectiveness of the interpolation methods.

$$ErrorTerm = \sqrt{\sum((P(x) - Q(x))^2 | x \in T)}$$

3. The Result

3.1 Baseline Result

In this section, we will discuss the results for the baseline case. In the baseline case, the portfolio size is 30. The moneyness band is (1, 1.4). Assuming the uniform distribution, we pick 30 samples from the band as the strike of the options in the portfolio. The maturity band is (0.25, 2.25).Assuming the uniform distribution, we pick 30 samples from the band as the maturity of the options in the portfolio. The sample Break is set at (0.05, 0.50, 0.95). The fitting portfolio will have 9 options. The maturity and the strike of 9 options are combination of the 3 choices of maturity at the 5th percentile, 50th percentile and 95th percentiles of the 30 maturities in the portfolio, and 3 choices of moneyness at the 5th percentile, 50th percentile and 95th percentiles of the 30 strikes in the portfolio.

The following table summarizes the basic information of the portfolio, and the outcome of the four interpolation methods. The current spot price is $100. The current portfolio value is the sum of price of 30 call options which is $106.93. If the spot moves up 20%, the portfolio value could become $339.05. The market moves down 20%, the portfolio would become $18.70.

The Polynomial Fit produced an Error term of $0.0061 and the Cubic Spline fit produced an Error term of 0.89830. The Black Sholes Inverse methods, where the 9 options in the fitting portfolio are permitted to be both long and short, produced Error term of 0.0004, which is better than the Polynomial fit. The quantity of each of the 9 options in the fitting portfolio varies. The maximum (long) quantity (notional) in the fitting portfolio is $96.78. The minimum (short) quantity (notional) of the option is ($145.74). Compared to the portfolio value, the quantity (notional) looks reasonable.

In the Black SholesFit method, we define the goal function as the square root of the sum of squares of the difference of P(x) and Q(x) where x is in set S. We will optimize the quantity (notional) to minimize the goal function. The goal function = 0 if and only ifP(x)=Q(x)for x in S. After optimization, the goal function had a minimum of 2.47. As the fitting portfolio contains "long" options only, the fitting portfolio failed to match the original portfolio at the 9 pivot points. The Error term is 4.93231, which is not desirable.

Tabe 1 One Trial of the Baseline Assumption Moneyness (1, 1.4), Maturity (0.25, 2.25), Sample Break (.05, 0.5, 0.95), Portfolio Size 30

	Summary of Portfolio Sensitivity			Error using Different Methods				Other Information			
Case #	Current Portfolio Value	Portfolio Value +20% Shock	Portfolio Value -20% Shock	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
1	106.93	339.05	18.70	4.93231	0.89830	0.00061	0.00004	2.47	15.72	96.78	(145.74)

3.2 Sensitivities results

In this section, we will discuss the effectiveness of the interpolation methods for various characteristics of the portfolio construction such as portfolio size, span of the maturities and span of strikes.

First, we study the stability of the performance of different interpolation methods using a simulation approach. Under the baseline assumption of moneyness (1, 1.4),maturity (0.25, 2.25),sample break (.05, 0.5, 0.95),portfolio size 30, we run 20 simulations of different portfolios.

Among the three methods of matching the price at all 9 pivot points, the Black Sholes-Inverse is better than the Polynomial Fit for all 20 runs except the Case #13. In Case #13, Black Sholes-Inverse produces irregular results as the inverse of the 9x9 matrix would produce huge numerical values due to the determinant of the matrix is close to zero. The row vectors or column vectors of the matrix are highly correlated. Choosing a wider sample break points at (0.05, 0.5, 0.95) would reduce likelihood of this happening, but it will not eliminate it(see Table 2 below). In case #13, the fitting coefficient is multiples of the portfolio value (highlighted in yellow), and the Error of the Black Sholes-Inverse (0.04356) is also larger than the Error of the Poly-Fit (0.00233).The Cubic Spline is the consistently worst of the three methods. The Error term is at a different magnitude.

In the Black Scholes-Fit method, the fitted portfolio consists of 9 long options, with no short options. The valuation at the 9-pivot point does not match the original portfolio value no matter how many iterations were run. The square sum of the difference is in the column "Black Sholes-Fit Goal."TheError using the Black Sholes-Fit will be larger when the Black Sholes-Fit Goal is larger.

The relative performance of the 4 methods is stable. The Black Sholes-Inverse is the best unless there isan irregularity in the matrix inversion. The Polynomial-Fit is the second best, the Cubic Spline the third. The Black Sholes-Fit is the worst of all methods due to the restriction of long options only.

Tabe 2 Baseline Assumption Moneyness (1, 1.4), Maturity (0.25, 2.25), Sample Break (.05, 0.5, 0.95), Portfolio Size 30

	Summary of Portfolio Sensitivity			Error using Different Methods				Other Information			
Case #	Current Portfolio Value	Portfolio Value +20% Shock	Portfolio Value -20% Shock	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
1	106.93	339.05	18.70	4.93231	0.89830	0.00061	0.00004	2.47	15.72	96.78	(145.74)
2	86.68	305.93	12.45	6.07243	0.95940	0.00046	0.00001	3.03	24.05	40.97	(59.26)
3	108.29	352.12	16.95	1.68288	0.91643	0.00035	0.00002	0.84	21.41	205.58	(149.91)
4	107.41	337.16	20.74	11.98954	1.00653	0.00079	0.00025	6.02	14.47	56.26	(121.72)
5	106.40	343.73	17.64	5.33372	0.89681	0.00019	0.00005	0.27	20.57	98.71	(128.30)
6	85.54	307.08	12.01	6.98160	0.95834	0.00069	0.00006	3.48	24.31	41.74	(49.57)
7	96.85	322.96	15.55	6.32065	0.92343	0.00124	0.00012	3.16	21.32	102.50	(113.05)
8	78.83	285.50	11.20	8.17059	0.92114	0.00202	0.00037	4.07	25.03	57.99	(10.81)
9	117.31	373.92	20.21	3.28951	0.87383	0.00134	0.00037	1.60	17.74	142.80	(150.06)

10	88.36	300.79	12.79	0.56141	0.84046	0.00042	0.00003	0.29	19.97	34.62	(20.25)
11	94.46	324.72	15.58	10.46098	0.99526	0.00065	0.00012	5.23	22.46	84.51	(138.28)
12	114.94	383.97	18.03	3.50000	0.94914	0.00325	0.00031	1.68	23.64	163.77	(209.53)
13	122.86	391.10	21.03	7.04095	0.93429	0.00233	0.04356	3.49	21.89	1,801.83	(7,156.03)
14	87.42	312.89	12.98	7.75784	1.02725	0.00145	0.00026	3.87	25.24	16.54	(9.95)
15	105.40	348.63	16.51	1.76697	0.93438	0.00027	0.00001	0.89	19.84	31.55	(66.72)
16	103.04	337.79	16.58	0.80932	0.86711	0.00036	0.00001	0.41	17.79	29.25	(39.49)
17	107.33	350.48	18.04	7.52644	0.93327	0.00122	0.00056	3.77	22.38	35.03	(86.29)
18	92.39	327.48	13.69	9.50934	0.96685	0.00284	0.00047	4.72	25.65	107.50	(98.61)
19	111.25	360.40	18.63	4.23729	0.89887	0.00304	0.00064	2.11	20.09	180.55	(264.63)
20	111.85	356.99	19.04	3.93741	0.88765	0.00229	0.00022	1.97	18.89	82.24	(109.84)
Percentile											
95%	117.59	384.33	20.75	10.53741	1.00757	0.00305	0.00279	5.27	25.26	285.39	(10.77)
75%	109.03	353.34	18.65	7.58429	0.95861	0.00209	0.00037	3.80	23.74	116.33	(56.84)
50%	105.90	338.42	16.77	5.70308	0.92835	0.00101	0.00017	2.75	21.37	83.38	(111.45)
25%	91.38	320.44	13.51	3.44738	0.89793	0.00045	0.00004	1.42	19.60	39.49	(146.78)
5%	85.20	300.03	11.97	0.79692	0.86578	0.00027	0.00001	0.29	15.66	28.61	(609.20)

The following Table 3 has the sensitivity to the sample break points in terms of the maturity and the moneyness of the options portfolio. As expected, the narrower break points, with a more linearly dependent column / row vectors of the matrix, makes the matrix inverse produce irregular results. We can see the explosion of the weights in the Black Sholes Inverse approach. The Black Sholes-Fit does not work well neither.

Tabe 3Sensitivity to Sample Break points, Baseline Assumption Moneyness (1, 1.4), Maturity (0.25, 2.25), Portfolio Size 30

	Sample Break Points			Error using Different Methods				Other Information			
Case #	Break 1	Break 2	Break 3	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
1	0.05	0.50	0.95	1.03290	0.92705	0.00084	0.00009	0.53	23.17	32.04	(48.44)
2	0.15	0.50	0.85	33.21797	0.92705	0.00084	780.34368	0.71	20.81	267.21	(347.65)
3	0.25	0.50	0.75	93.77078	0.92705	0.00084	3,723.36639	1.19	16.68	739.63	(579.63)

Table 4 has the sensitivity to the Portfolio size. It appears the performance of the four methods are similar regardless of the portfolio size. Note that the Black Sholes-Inverse Max / Min weight both increases in similar magnitude to the portfolio value, and the Error term using the Black Sholes-Inverse remained smaller than the other three methods.

Tabe 4 Sensitivity to Portfolio Size, Baseline Assumption Moneyness (1, 1.4), Maturity (0.25, 2.25), Sample Break (.05, 0.5, 0.95)

	Summary of Portfolio Sensitivity			Error using Different Methods				Other Information			
Portfolio Size	Current Portfolio Value	Portfolio Value +20% Shock	Portfolio Value -20% Shock	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
10	44.80	154.07	5.05	5.35708	0.29802	0.00084	0.00002	2.69	11.30	35.99	(30.35)
30	114.25	379.88	17.14	1.03290	0.92705	0.00084	0.00009	0.53	23.17	32.04	(48.44)
100	363.92	1,194.66	58.05	9.30332	3.06691	0.00121	0.00018	4.62	67.85	80.73	(134.82)
250	864.81	2,892.21	135.82	35.35546	7.79132	0.00068	0.00009	17.58	176.54	325.63	(491.36)

Table 5 has the sensitivity to the moneyness range. When the moneyness moves out of money, the value of the portfolio is reduced. The sensitivity of the portfolio to market shocks is also reduced. The Black Sholes-Inverse approach tends to produce irregular results. In the case of moneyness Band (1.2, 1.6), The Black Sholes-Inverse Max weight is $2012.92, vs that the Portfolio value is only $24.24. The other three methods seem to perform normally.

Tabe 5Sensitivity to Moneyness, Baseline Assumption Maturity (0.25, 2.25), Sample Break (.05, 0.5, 0.95), Portfolio Size 30

Moneyness Range			Error using Different Methods				Other Information			
Current Portfolio Value (of one trial)	Lower Bound of Moneyness	Upper Bound of Moneyness	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
114.25	1.00	1.40	1.03290	0.92705	0.00084	0.00009	0.53	23.17	32.04	(48.44)
45.09	1.10	1.50	4.14796	0.82757	0.00086	0.00029	2.05	18.45	278.38	(681.50)
24.24	1.20	1.60	2.16404	0.71435	0.00018	0.00011	1.06	21.07	2,012.92	(955.66)

Table 6 has the sensitivity to maturity of the option. It appears the performance of the four methods are similar regardless of maturity band, when maturity band does not impact the portfolio value significantly.

Tabe 6Sensitivity to Maturity, Baseline Assumption Moneyness (1, 1.4), Sample Break (.05, 0.5, 0.95), Portfolio Size 30

Moneyness Range			Error using Different Methods				Other Information			
Current Portfolio Value (of one trial)	Lower Bound of Maturity	Upper Bound of Maturity	Error using BS-Fit	Error using Cubic Spline	Error using Poly-Fit	Error using BS-Inverse	BS-Fit Goal	BS-Fit Max Weight	BS Inverse Max Weight	BS Inverse Min Weight
114.25	0.25	2.25	1.03290	0.92705	0.00084	0.00009	0.53	23.17	32.04	(48.44)
136.98	0.75	2.75	2.52362	0.82529	0.00004	-	1.28	19.80	15.73	(8.64)
187.03	1.50	3.50	2.91157	0.76677	0.00002	0.00002	1.51	20.26	20.10	(4.99)

4. Conclusions and Further Research

The Black Sholes-Inverse method is the best method except when the inverse matrix produces an irregular result. The irregularity can be spotted by looking at the magnitude of the weight relative to the current portfolio valuation. The Polynomial Fit method performed consistently. The Cubic Spline is less effective than the Polynomial Fit. Finaly, the Black Sholes-Fit is less optimal as the restriction of long options only limited the possibility of matching the price even at the pivot points.

Interpolation question is one of the basic questions in calculus or analysis of functions. The expansion of the functions such as Taylor expansion and Fourier expansion is a systematic attempt to perform the interpolation, using the power function and the sine cosine function for the periodic functions. As the Black Sholes-Inverse produces a better fit than the Polynomial Fit, it is shown that the Black-Scholes function is a more appropriate function for interpolation. When the portfolio consists of European options, Black-Sholes formula is a better function for the interpolation. Will this be the case for other financial instruments? More generally, as a pure mathematical question, would we define / classify these types of functions?

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