American Review of Mathematics and Statistics
December 2023, Vol. 11, No. 2, pp. 1-16
ISSN: 2374-2348 (Print), 2374-2356 (Online)

Published by American Research Institute for Policy Development
DOI: 10.15640/arms.v11n2a1
URL: https://doi.org/10.15640/arms.v11n2a1

Random Matrix Theory Analysis of Bank Stocks in Nigeria and Use of Implied Volatility in Risk Management

T.C. Urama[1]

Abstract

This paper examines cross-correlation matrix C of stock index returns obtained from Nigerian banking sector for the period 2009 to 2021 using the concept of Random Matrix Theory. The eigenvalues of the empirical correlation matrix are tested and their respective eigenvectors used to determine which bank(s) that drive the financial sector of the Nigerian Stock Market (NSM) through an analysis of their inverse participation ratios. It was observed that there are predominantly positive correlation among the respective stocks, meaning that individual stocks move in the same direction, hence diversification of assets in the banking sector may not be an optimal strategy except for Unity and Union bank stocks that have negative correlation with most of other bank stocks. To this end, and taking cognizance of the fact that Nigeria is yet to commence full trading on derivative products, hence no data yet on derivative trade in Nigeria, we try to estimate the realistic implied correlation matrix from some hypothetical option prices for some assets listed in the NSM as demonstration of ways to diversify assets through the use of derivatives. Finally, in this work we derive Marcento-Pastur law in the appendix.

Key words: Random Matrix Theory, empirical correlation matrix, eigenvalues and vectors, realistic implied correlation matrix and implied volatility.

1.0 Introduction

We examine the spectral properties of the correlation matrix of the price variations in Nigerian Stock Market (NSM), by scrutinizing the dynamics of bank stocks price movement and trends in the fluctuations, using the Random Matrix Theory (RMT). We investigate the correlation matrix using RMT, through a comparison of the empirical correlation matrix with that of the Wishart random matrix. The linear relationships among assets in a given market is usually summarized in a correlation matrix hence the need to study RMT in any financial market(s) of interests.

Our proposal to deal with data of high dimensionality in the stock price returns is through the study of Random Matrix Theory which has become a good tool employed in this field that seeks to adopt the method of the interactions' of the nuclei of complex atoms [1] to determine the true structure of financial markets [2]. RMT methods for analysis of the properties of correlation matrix (C), shows that 98% of the eigenvalues of C agree with RMT predictions suggesting an appreciable degree of randomness in the measured cross correlation [3]. They assert that there are deviations from RMT predictions for 2% of the largest eigenvalues and that the largest eigenvalue of C represent the influence of the entire market that is common to all stocks.

Szilard Pafka and ImreKondor [4] assert that correlation matrices of financial returns play a crucial role in various aspects of modern finance including investment theory, capital allocation and risk management. In their view, for a theoretical perspective, the main interest in examining correlation of price returns is for proper description of the structure and dynamics of correlations whereas for a practitioner, the emphasis is on the ability of the models to provide adequate inputs towards the numerous portfolios and risk management procedures required in the financial industry.

[1] Department of Statistics, Institute of Management and Technology (IMT) Enugu, Campus 3, P.M.B 01079, Independence Layout Enugu - Enugu State, Nigeria, Emails: chinweurama@imt.edu.ng; chinweurama@yahoo.com
Phone: +2348035320568 or +2348168713309

Kawee Numpac haroen [5] observes that financial institutions usually hold multiple assets in their portfolios that may include basket of options or derivatives, credit derivatives or other correlation trading products which depend largely on the correlation coefficients between the underlying assets, hence the need to study RMT.

Sensoy, A. et al.[6] affirm that high correlation among stocks in any portfolio of assets means that the benefits of portfolio diversification is lowered since according to their finding, high correlation is synonymous to high volatility of stock prices. In this situation therefore, the better alternative for investors is thus thinking through the derivative (option) trade as a profitable risk management process in their portfolio of investments. Therefore, it becomes imperative that one should carry out a comprehensive analysis of the nature of correlation among assets in any given financial market and thereafter relate the observed stock price dynamics and the information therein as a useful tool in the hand(s) of investors in such markets. It is worthy of mention that following the introduction of RMT into the financial markets by R.N. Mantegna [7], Laloux et al. [8] and Plerou et al. [9], RMT has been found to be indispensable in study of statistical properties and stock price dynamics of cross-correlation in different financial markets [10-35].

[8] declare that for financial assets, banks inclusive, the study of empirical correlation matrix is very important, and from their investigation, the estimation of the correlations between the price movements of different assets constitutes an important and indispensable aspect of risk management. They indicate that the likelihood of large losses for a certain portfolio or option book is dominated by correlated moves of its various constituents and that a position which is simultaneously short in bonds and long in stocks or vice-versa will be perilous since bonds and stocks usually move in reverse directions, especially during crisis periods. When the asset diversification approach to risk management fails as a result of a very high correlation among stocks, investors in the given financial market are required to use derivatives products as a hedge on the underlying assets for risk management and are, consequently encouraged to buy call/put options respectively for those assets whose returns move in opposing directions as may be inferred from the calculated empirical correlation matrix. Furthermore, Plerou et al.; [15] opine that an accurate quantification of correlations between the returns of various stocks is of practical importance in determining the risk of portfolios of stocks, pricing of options and forecasting. They declare that financial correlation matrices are the salient input parameters to Markowitz's fundamental theory of portfolio optimization problem [36] that aims at providing a recipe for the selection of a portfolio of assets so that the risk associated with the investment is minimized for a given expected return.

The fascinating question that concerned investors need to answer is how the (implied) volatility, which is a measure of market fluctuations, affects the dynamics of the market or vice versa. It is, therefore, beneficial to examine the relationship between the two distinct properties of market which includes volatility as an index of market fluctuations and the coupling of stocks with one another using the concept of correlation matrix [22]. Correlations amongst the volatility of different assets are very useful, not only for portfolio selection, but also in pricing of options and certain multivariate econometric models for price forecasting and volatility approximations [37]. They contend that with regards to Black-Scholes [38] option pricing model the variance of portfolio, ϱ of options exposed only to Vega risk is given by

$$Var(\varrho) = \sum_{i,j,k,l} \frac{w_i w_l \Lambda_{ij} \Lambda_{lk} C_{jk}}{v_j v_k \sigma_j \sigma_k} \tag{1}$$

where w_i are the weights in the portfolio, C_{ij} is the correlation matrix for the implied volatility for the underlying assets and the Vega matric Λ_{ij} is defined as

$$\Lambda_{ij} = \frac{\partial p_i}{\partial v_j} \tag{2}$$

with p_i as the price of option i, v_j is the implied volatility of asset underlying option j and σ_i is the standard deviation of the implied volatility v_i.

It our goal to evaluate the correlation microstructure of the stock price dynamics particularly for the bank assets enlisted in the Nigerian Stock exchange as against the earlier study of the entire stocks listed in the NSM considered by Urama, et al. [39]. This will, no doubt, provide useful hints for investors in the Nigerian Market taking in lieu of the fact that derivative products are being introduced to the NSM as a tool for risk management to investors and also remembering that next to the oil industry the major assets that have overbearing influence in the Nigerian Market is the banking sector.

Edelman Alan [40] advocates the use of random matrix theory properties as a juxtaposition between the cross-correlation matrices obtained from a given number of empirical time series of underlying stocks data for a period T with an absolutely random Wishart matrix W, of the same size with the empirical correlation matrix in order to obtain some useful information about the market(s) to be deployed in portfolio optimization and risk management. RMT predictions represent the mean of all possible interactions between the constituent assets in a given market under consideration. The departure of the eigenvalues from universal predictions of RMT obtained from the Wishart matrix is used in identifying the system specific, non-random properties of the market under consideration and such deviations usually about 2% [3] provide information about the underlying interactions of the assets. The absence of deviating eigenvalues in the region predicted by RMT means that the entire system is engulfed by noise (is random), hence no statistical inference could be drawn from the analysis.

In other words, the process is to compare the statistics of the cross-correlation coefficients of price fluctuations of stock i and j against a random matrix of the same size, having the same symmetric properties as that of the empirical matrix. The RMT is known to distinguish the random and non-random parts of the cross-correlation matrix C and the non-random parts of C which deviates from RMT results is known to provide information regarding genuine collective behaviour of the stocks under consideration and indeed the entire market from where the sample stocks were drawn [16].

The investigation of correlations among price changes of various assets in a given market is not only necessary for quantifying the risk in a given portfolio but also of scientific interest to researchers in economics and financial mathematics [42,43]. Nonetheless, the problem of interpreting the correlations between individual stocks-price changes in a given financial market can be likened to the difficulties experienced by physicists in the fifties, in interpreting the spectra of complex nuclei. Due to the huge amounts of spectroscopic data on the energy levels that were available which were too complex to be interpreted through model calculations, since the nature of the interactions were not known, the concept of Random Matrix Theory (RMT) was developed to take care of the statistics of energy levels of the complex quantum systems [43-45].

Analogously, for financial time series in a stock exchange, the nature of interactions among constituent stocks are unknown hence the need to adopt the RMT method in explaining the influence each individual stock has with the others within the same market. This, no doubt, will provide the desired market microstructure of stock price dynamics desired for portfolio optimization and risk management. It is, therefore, this estimation of risk and expected returns, based on variance and expected returns in a given portfolio that constitutes Markowitz's model [36].

In carrying out RMT method of portfolio optimization and risk management, the period T, under consideration, has to be relatively large in comparison with the number of stocks being considered in order to minimize the noise in the correlation matrix. The two sources of noise envisaged in the use of RMT to investigate the dynamics of cross-correlations of stocks in a given financial market include: the noise from the period length T considered with respect to the number of stock and that emanating from the fact that financial time series of historical return itself is finite or bounded, thus introducing, inadvertently estimation errors (noise) in the correlation matrix [4]. Szilard and Kondor [47] also discover that the effect of noise strongly depends on the ratio $= \frac{N}{T}$, where N is the number of stocks considered and T the length of the available time series. They discover that for the ratio r = 0.6 and above, there will be a remarkable effect of noise on the empirical analysis, as was also asserted by G. Galluccio et al. [48]; V. Plerou et al. [4]; L. Laloux et al. [34] and they infer that for smaller value of r ($r = 0.2\ or\ less$); the error due to noise drops to an admissible level(s). For this research, we use the empirical data obtained from NSM, with $r = \frac{15}{3101} = 0.005 < 0.2,$ thus within the tolerable value of r.

In the following analysis, if the obtained eigenvalues of the empirical correlation matrix and that of the Wishart matrix lie in the same region without any significant deviations, then the stocks are said to be uncorrelated and therefore no inference or deduction can be made about the nature of the market. However, if on the contrary there exists at least one eigenvalue lying outside the theoretical bound of the eigenvalues in the empirical correlation matrix obtained from the stock market returns with that of the theoretical Wishart matrix, then the deviating eigenvalue(s) is(are) known to carry information about the market under consideration, and the asset whose component corresponds with the leading deviating eigenvalue is said to drive the financial market and indeed in this case the stock driving the banking industry in Nigeria.

2. Data

The Data set is made up of the daily closing prices of 15 bank stocks listed in the Nigerian Stock Market, NSM from 3rd August 2009 to 31st December, 2021, giving a total of {3101} daily closing returns after removing bank assets that were delisted, that did not trade at all or that have partially traded for few days within the period under review. The bank stocks considered are Access Bank, Diamond (Access) Bank, Equatorial Trust Bank otherwise known as Ecobank, First Bank of Nigeria, First City Monument Bank, Fidelity Bank, Guaranty Trust Bank, Skye Bank now Polaris Bank, Stanbic IBTC Bank, Sterling Bank, United Bank for Africa, Union Bank, Unity Bank, WEMA Bank and Zenith Bank.

We remark that for the daily asset prices to be continuous and to minimize the effect of thin trading, it is, therefore, expedient to remove the public holidays in the period under consideration. Furthermore, to reduce noise in the analysis, market data for the present day is assumed to be the same with that of the previous day in the case(s) where there is no information on trade for any particular asset on a given date(s).

Let $P_i(t)$ be the closing price on a given day t, for stock i and define the natural logarithmic return of the index as

$$r_i(t) = \ln\frac{P_i(t+1)}{P_i(t)} \qquad (3)$$

where$r_i(t)$ is the number of observations in the Nigerian Stock Market (Bank Stocks).

3 Theoretical Backgrounds

3.1 Computing Volatility

We compute price changes in assets over a time scale Δt which is equivalent to one day and denote the price of $(x+a)^n = \sum_{k=0}^{n}\binom{n}{k}x^k a^{n-k}$ at a time t as $s_i(t)$ with the corresponding price change or logarithmic returns $G_i(t)$ over time scale Δt as

$$G_i(t) = \ln\,[S_i(t+\Delta t)] - \ln\,[S_i(t)] \qquad (4)$$

Next, we quantify volatility in the respective asset return as a local average of the absolute value of daily returns of indices in an appropriate time window of T days as

$$v = \frac{\sum_{t=1}^{T-1}|G_i(t)|}{T-1} \qquad (5)$$

To standardize the values obtained from equation (4) above for all values of i, we normalize $G(i)_t$ as follows:

$$g(t)_i = \frac{G(t)i - \langle G(t)i\rangle}{\sigma_i} \qquad (6)$$

where$\sigma_i = \sqrt{\langle G(t)_i^2\rangle - \langle G(t)_i\rangle^2}$ $and\ \langle\ldots\rangle$ represents the average in the period studied.

From real time series return data, we can obtain the elements of $N\ x\ N$ correlation matrix C as follows

$$C_{ij} = \langle g_i(t)g_j(t)\rangle = \frac{\langle[G_i(t) - \langle G_i\rangle][G_j(t) - \langle G_j\rangle]\rangle}{\sqrt{[\langle G_i^2\rangle - \langle G_i\rangle^2][\langle G_j^2\rangle - \langle G_j\rangle^2]}} \qquad (7)$$

C_{ij} lies in the closed interval $-1 \le C_{ij} \le 1$, with$C_{ij} = 0$ means there is no correlation, $C_{ij} = -1$ implies anti-correlation and $C_{ij} = 1$ means perfect correlation for the empirical correlation matrix gotten from implied volatility surface.

3.2 Eigenvalue spectrum of the correlation matrix

As specified earlier, our aim is to extract information about the cross correlation from the empirical correlation matrix C. To this end, we compare the properties of C with those of a random matrix, T. Colon et al. [49]; L. laloux et al. [34]; V. Plerou et al. [9]; P. Gopikrishnan et al. [35]; V. Plerou et al. [17]. It can be shown from Sharifi, S [11] that the empirical correlation matrix C can be expressed as

$$C = \frac{1}{L}GG^T \qquad (8)$$

where G is the normalized N x L matrix andG^T is the transpose of G. This empirical correlation will be compared with a random Wishart matrix (random matrix) R given by:

$$R = \frac{1}{L}AA^T \qquad (9)$$

so as to distinguish the information from noise in the system, T. Colon et al. [49] and P. Gopikrishnan et al. [35], where A is an $N\ x\ L$ matrix whose entries are independent identically distributed random variables that are normally distributed and have zero mean and unit variance.

In our effort to use the random matrix theory in portfolio optimization and (derivative) assets risk management, we should be conversant with the universal properties of random matrices. Wilcox et al. [31] contend that there are four underlying properties of random matrices which include:

(a) Wilshart distribution of eigenvalues from the correlation matrix, (b) Wigner surmise for eigenvalue spacing (c) the distribution of eigenvector components of the corresponding eigenvalues and finally (d) Inverse participation ratio for eigenvector components of the resulting correlation matrix.
[50-52], and A. Edelman [40] assert that the statistical properties of R are known and that in particular for the limit as $N \to \infty, and\ L \to \infty$ we have that $Q = \frac{L}{N}(\geq 1)$ is fixed and that the probability function $P_{rm}(\lambda)$ of eigenvalues λ of the random correlation matrix R is given by

$$P(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{max}-\lambda)(\lambda-\lambda_{min})}}{\lambda} \tag{10}$$

for λ such that $\lambda_{min} \leq \lambda \leq \lambda_{max}$, where σ^2 is the variance of the elements of A. Here $\sigma^2 = 1$ and $\lambda_{min}\ and\ \lambda_{max}$ satisfy

$$\lambda_{max/min} = \sigma^2(1 + \frac{1}{Q} \mp 2\sqrt{1/Q}) \tag{11}$$

The values of lambda from equation (11) that satisfy (12) and (13) are called the Wishart distribution of eigenvalues from the correlation matrix. These values of lambda as stated earlier determine the bounds of theoretical eigenvalue distribution. When the eigenvalues of empirical correlation matrix C are beyond these bounds, they are said to deviate from the random matrix bounds and are therefore supposed to carry some useful information about the marketstocks under consideration [28].

The distribution of eigenvalue (surmise of eigenvalue) spacing was introduced as the required test for the case when there are not significant deviations of the empirical eigenvalue distribution to that of the random matrix prediction, [31]. When the eigenvalues so obtained from the correlation matrix do not deviate significantly from the predictions of the RMT we apply the so-called Wigner surmise for eigenvalue spacing otherwise called Gaussian orthogonal ensemble Plerou et al. [17] and is given by

$$P(s) = \frac{s}{2\pi}\exp\left(-\frac{s\pi^2}{4}\right), \tag{12}$$

where $s = (\lambda_{i+1} - \lambda_i)/d$ and d denotes the average of the differences $\lambda_{i+1} - \lambda_i$ as i varies.

3.3 Distribution of eigenvector component

The concept that low lying eigenvalues are really random can also be verified by studying the statistical structure of corresponding eigenvectors. The jth component of the eigenvector corresponding to each eigenvalue λ_α will be denoted by $v_{\alpha,j}$ and then normalized such that $\sum_{j=1}^{N} v_{\alpha,j}^2 = N$. Plerou et al. [9] assert that if there is no information contained in the eigenvector $v_{\alpha,j}$, one expects that for a fixed α, the distribution of $u = v_{\alpha,j}(as\ j\ is\ varied)$ is a maximum entropy distribution. This therefore leads to what is called Porter-Thomas distribution in the theory of random matrices written as

$$p(u) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right) \tag{13}$$

It has been found that the eigenvector components $v_{\alpha,j}$ for α=1, 2, 3 ….n of an eigenvector v_α are normally distributed with zero mean and unit variance, [53]. The distribution so obtained from (13) above are expected to fit well the histogram of the eigenvector except for those corresponding to the highest eigenvalues which lie beyond the theoretical value of λ_{max}, [9].

3.4 Inverse participation ratio

Guhr, T. et al. [53] assert that in order to quantify the number of components that participates significantly in each eigenvector, we use inverse participation ratio. Inverse participation ratio (IPR) shows the degree of deviation of the distribution of eigenvectors from RMT results and distinguishes one eigenvector with approximately equal components with another that has a small number of large components. For each eigenvector v_a, V. Plerou et al. [17] defined the inverse participation ratio as

$$I_\alpha = \sum_{j=1}^{N} (v_\alpha^j)^4 \quad (14)$$

where N is the number of the time series (the number of implied volatility considered) and hence the number of eigenvalue components and v_α^j is the j th component of the eigenvector v_α. There are two limiting cases of I_α (i) when an eigenvector v_α has an identical component, $v_\alpha(j) = \frac{1}{\sqrt{N}}, then\, I_\alpha = \frac{1}{N}$ and (ii) For the case wherev_α has one element with $v_\alpha(j) = 1$ and the remaining components zero, then $I_\alpha = 1$.

Therefore, the IPR can be illustrated as the inverse of the number of elements of an eigenvector that are different from zero that contribute significantly to the value of the eigenvector. A. Utsugi et al. [54] in their study of the RMT declare that the expectation of the IPR is given by

$$\langle I_\alpha \rangle = N \int_{-\infty}^{\infty} [\, v_\alpha(j)]^4 \frac{1}{\sqrt{2\pi N}} \exp\left(-\frac{[v_\alpha(j)]^2}{2N}\right) dv_\alpha(j) = \frac{3}{N} \quad (15)$$

since the kurtosis for the distribution of eigenvector components s 3.

4.0 Empirical Result and data analysis

	Access	Diamond	ETI	FBN	FCMB	Fidelity	Guaranty	Skye Bank	Stanbic	Sterling	UBA	Union	Unity	WEMA	Zenith
Access	1	0.3125	0.3034	0.2754	0.1824	0.3331	0.1845	0.2121	0.2204	0.1322	0.2564	-0.0254	0.0542	0.0512	0.4212
Diamond	0.3125	1	0.2234	0.2542	0.2374	0.4111	0.2635	0.2101	0.2234	0.2154	0.2182	-0.0122	0.0754	0.173	0.2287
ETI	0.3034	0.2234	1	0.2133	0.0987	0.2111	0.2287	0.1487	0.1153	0.2252	0.2314	0.0126	0.0312	0.1186	0.2545
FBN	0.2754	0.2542	0.2133	1	0.0966	0.2354	0.3444	0.1714	0.1292	0.1341	0.3831	-0.049	0.032	0.1811	0.3528
FCMB	0.1824	0.2374	0.0987	0.0966	1	0.2531	0.1974	0.3245	0.2241	0.1987	0.1466	-0.0214	0.0921	0.022	0.2515
Fidelity	0.3331	0.4111	0.2111	0.2354	0.2531	1	0.2736	0.1958	0.0256	0.1598	0.2659	0.0521	0.1266	0.1354	0.2522
Guaranty	0.1845	0.2635	0.2287	0.3444	0.1974	0.2736	1	0.1271	0.2145	0.2087	0.1853	0.0132	-0.008	0.0969	0.3214
Skye Bank	0.2121	0.2101	0.1487	0.1714	0.3245	0.1958	0.1271	1	0.1244	0.1374	0.2414	-0.0537	0.0941	0.2018	0.1533
Stanbic	0.2204	0.2234	0.1153	0.1292	0.2241	0.0256	0.2145	0.1244	1	0.0521	0.1461	-0.0247	0.0159	0.0941	0.131
Sterling	0.1322	0.2154	0.2252	0.1341	0.1987	0.1598	0.2087	0.1374	0.0521	1	0.045	-0.0245	0.089	0.0854	0.2654
UBA	0.2564	0.2182	0.2314	0.3831	0.1466	0.2659	0.1853	0.2414	0.1461	0.045	1	-0.1541	0.1254	0.0722	0.4587
Union	-0.0254	-0.0122	0.0126	-0.049	-0.0214	0.0521	0.0134	-0.0537	-0.0247	-0.0245	-0.1541	1	0.0521	-0.0587	-0.0544
Unity	0.0542	0.0754	0.0312	0.032	0.0921	0.1266	-0.008	0.0941	0.0159	0.089	0.1254	0.0521	1	0.1501	0.0533
WEMA	0.0512	0.173	0.1186	0.1811	0.0622	0.1354	0.0969	0.2018	0.0941	0.0854	0.0722	-0.0587	0.1501	1	0.2555
Zenith	0.4212	0.2287	0.2545	0.3528	0.2515	0.2522	0.3214	0.1533	0.131	0.2654	0.4587	-0.0544	0.0533	0.2555	1

Empirical correlation matrix for bank stocks in the NSM

4.1 Eigenvalue analysis

We took a sample study of 15 (N=15) bank stocks from the Nigerian stock market totaling L= 3101 daily closing prices and the theoretical eigenvalue bounds are respectively $\lambda_{-} = 0.2421$ and $\lambda_{+} = 1.3641$ as minimum and maximum values with $Q = \frac{L}{N} = \frac{3101}{15} = 206.73$. Further from the calculation the market value shows that the largest eigenvalue $\lambda_1 = 4.02$ which is approximately three times larger than the predicted RMT of value (1.36).

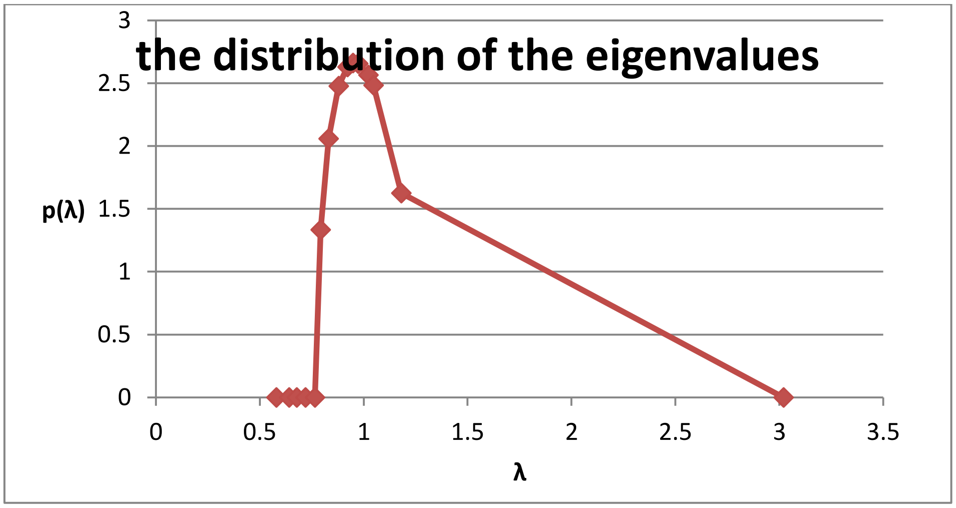

Figure 1: Theoretical (Marcenko-Pastur) empirical eigenvalues for banks in NSM.

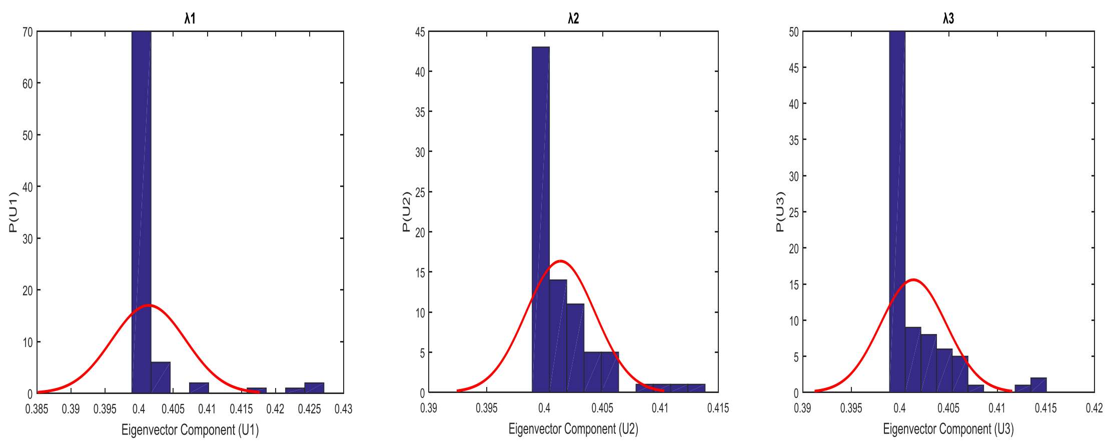

Figure 2: Distribution of eigenvector components of stocks in NSM

Figure 2 above represent the distribution of eigenvectors for the various eigenvalues in the empirical correlation matrix. The first diagram represents an eigenvector component for deviating eigenvalue in the theoretical region where as the other 2 are the eigenvector components of the eigenvalue within the regions predicted from the random matrix theory.

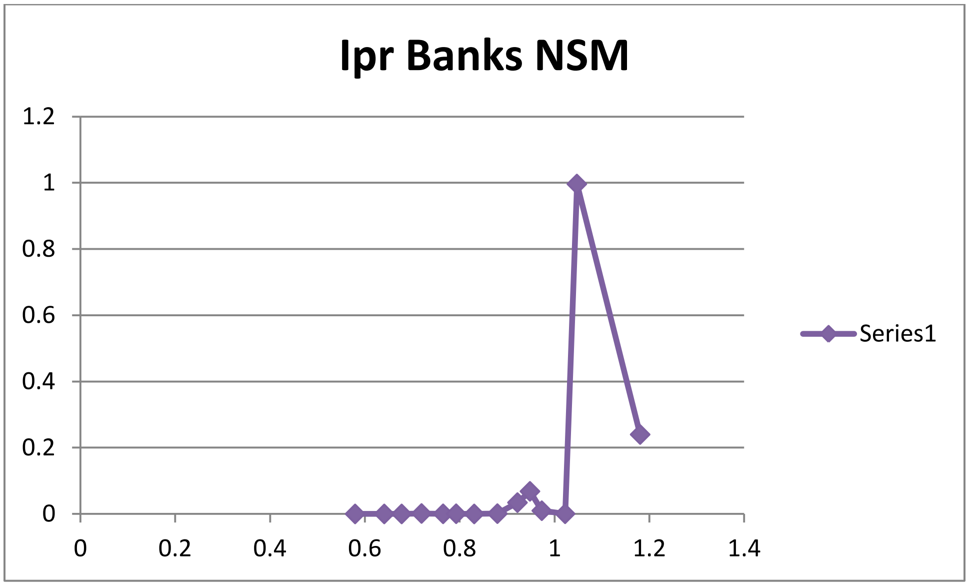

Figure 3: Inverse participation ratio and their ranks for NSM.

The inverse participation ratio (IPR) is the multiplicative inverse of the number of eigenvector components that contribute significantly to the eigenmode, [17]. For the largest eigenvalue (deviating from the RMT bounds) almost all the stocks contribute to the corresponding eigenvector thereby justifying treating this eigenvector as the market factor. The eigenvector corresponding to other deviating eigenvalues also exhibit that their corresponding stocks contribute slightly to the overall market features in the NSM. The average IPR value is around $^3/_{15}$ larger than would be expected $^1/_N = 0.07$, if all components contributed to each eigenvector, [53 former 49]. The remaining eigenvectors appear to be random with some deviations from the predicted value of $^3/_N = 0.2$ possibly as a result of the existence of fat tails and high kurtosis of the return distributions.

5. Conclusion and hints on future work

It was observed that 7 out of 15 bank assets considered that have their corresponding eigenvalues lie outside this theoretical bound of eigenvalues, therefore, 53% of the information from the return distributions is purely random thereby leaving us with the alternative hypothesis of the RMT which states that the information on the market lies on the deviating eigenvalues. This means then that for NSM banks the true market characteristic lies with a significant number of the stocks resulting to 47% of the banks considered.

It can be observed from the correlation matrix obtained that most of coefficients of each pairs have positive coefficients meaning that the respective stock move in the same direction hence the diversification method in the portfolio is not an optimal portfolio strategy. It is therefore better to invest in some derivative products like call and or put options to hedge against the risk on the portfolio in the Nigerian market.

Algorithm for Calculating Realistic Implied Correlation Matrix, R^Q

Kawee and NattachaiNumpacharoen [55] defined a valid empirical correlation matrix from an identity nxn matrix as a matrix with the following properties: (a) All the diagonal entries must be one which is the case for the empirical correlation matrix obtained from the sample of stocks considered with the NSM in this paper (b) Non-diagonal entries of C_{ij} are real numbers in the closed interval $-1 \leq C_{ij} \leq 1$ (c) The empirical correlation matrix is symmetric (d) The empirical correlation matrix must be positive (semi) definite to accommodate matrix decomposition for some desired purposes like Monte-Carlo simulation KaweeNumpacharoen [5]. They further stated that when the empirical correlation matrix are not identical as is the case with the matrix derived from the asset return distribution of stocks selected from NSM, the implied volatility of the portfolio σ_{port}^{Q} is given by

$$(\sigma_{port}^{Q})^2 = W * S^Q * C^Q * S^Q * W' \qquad (16)$$

Similarly, if $(\sigma_{port}^{Q}$ is the implied volatility of the portfolio obtained from C^P then it can be described as

$$(\sigma_{port}^{P})^2 = W * S^Q * C^P * S^Q * W' \qquad (17)$$

where, $W = [w_1 \quad \cdots \quad w_n]$ are the weights of the respective stocks in the portfolio;

$S^Q = \begin{bmatrix} \sigma_1^Q & 0 \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^Q \end{bmatrix}$ is a diagonal matrix obtained from the implied volatility of the respective assets being considered.

$C^Q = \begin{bmatrix} 1 & C_{2,1}^Q \cdots C_{n-1,1}^Q & C_{n,1}^Q \\ \vdots & \ddots & \vdots \\ C_{n,1}^Q & C_{n,2}^Q \cdots C_{n-1,n}^Q & 1 \end{bmatrix}$ is the desired realistic implied correlation matrix.

$$\text{or}(\sigma_{port}^Q)^2 = \sum_{i=1}^N \sum_{j=1}^N C_{i,j}^Q w_i w_j \sigma_i^Q \sigma_j^Q \quad (16a)$$

$$\rightarrow (\sigma_{port}^Q)^2 = \sum_{i=1}^N w_i^2 (\sigma_i^Q)^2 + 2\sum_{i=1}^{N-1} \sum_{j>i}^N C_{i,j}^Q w_i w_j \sigma_i^Q \sigma_j^Q \quad (16b)$$

Buss and Vilkov [56] assert that to identify $Nx(N-1)/2$ correlations that satisfy equation (16a), we propose the following parametric form for implied correlations:

$$C^Q = C^P - \varphi * (I_{nxn} - C^P) \quad (18)$$

where, C^Q is the expected correlation under the objective measure and φ is the parameter to be identified.
By substituting equation (18) into equation (16) we shall have:

$$\varphi = -\frac{(\sigma_{port}^Q)^2 - W * S^Q * C^P * S^Q * W'}{WS^Q(I_{nxn} - C^P)S^Q W'} \quad (19)$$

and from equations (16) and (17) is equivalent to:

$$\varphi = \frac{(\sigma_{port}^Q)^2 - \sum_{i=1}^N \sum_{j=1}^N C_{i,j}^P w_i w_j \sigma_i^Q \sigma_j^Q}{\sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_i^Q \sigma_j^Q (I_{nxn} - C^P)} \quad (20)$$

Buss and Vilkov [56] impose a restriction on the values φ to be in the region $-1 < \varphi \leq 0$for it to satisfy the technical conditions on the correlation matrix which includes that all the correlation $C_{i,j}^Q$ do not exceed one and that the correlation matrix is positive definite.

Now we consider the cases when $\varphi > 0$ and this occurs when $\sigma_{port}^P > \sigma_{port}^Q$ since$(I_{nxn} - C^P) \geq 0$ and this could make the realistic correlation matrix C^Q to be invalid when$\sigma_{port}^P < \sigma_{port}^Q$,[55]. He proposes a formula for valid correlation matrix that will take care of this shortcoming as stated below. Given any two valid correlation matrices C and D of dimensions nxn then there exists another valid correlation matrix F of the same dimension such that

$$F = w * C + (1 - w) * D \quad (21)$$

where$w$ is a real number in the interval $0 \leq w \leq 1.$

It therefore depends on the nature of the inequality existing between σ_{port}^P andσ_{port}^Q respectively that will inform our decision on the equivalent upper or lower bound equicorrelation matrix C to be used in obtaining a realistic implied correlation matrix. The corresponding equicorrelation matrices are represented by I_{nxn} for upper equicorrelation matrix and L_{nxn} matrix whose entries are $-\frac{1}{n-1}$ $for\ i \neq j\ and 1\ for\ i = j$ as the lower equicorrelation matrix.
Replacing equation F by C^Q and D by C^Pin (21) we will obtain

$$C^Q = C^P + w * (C - R^P) \quad (21a)$$

and from equations (16) and (17) we shall have:

$$w = \frac{(\sigma_{port}^Q)^2 - (\sigma_{port}^P)^2}{WS^Q(C - C^P)S^Q W'} \quad (22)$$

As a demonstration of the use of (yet to commence) derivatives trade in the NSM in managing portfolio of investment, we therefore consider some assets in NSM namely Guinness, Cadburys, Access Bank, AIICO Insurance, Zenith Bank and Nestle Foods. We want to show the use of realistic correlation matrix for risk management by assigning an arbitrary correlation matrix for the chosen assets drawn from the NSM with varied asset weight associated with the chosen stocks:

	Guinness	Cadburys	Access Bank	AIICO Ins	Zenith Bank	Nestle Foods
Guinness	1	-0.051	-0.003	0.003	-0.001	0.0157
Cadburys	-0.051	1	0.057	0.107	0.026	-0.001
Access Bank	-0.003	0.057	1	0.042	0.166	0.005
AIICO Insurance	0.003	0.107	0.042	1	0.027	0.001
Zenith Bank	-0.001	0.026	0.166	0.027	1	-0.007
Nestle Foods	0.017	-0.001	0.005	0.001	-0.007	1

Table1: A typical correlation matrix from NSM price return

Realistic Implied Correlation matrix computations:

Suppose we have the following weights and implied volatility (computed from option prices) for the under listed assets drawn from the Nigerian Stocks Market.

Asset	weight	implied volatility	Corr. Coffs.					
Guinness	15%	23%	1.0000	-0.0508	-0.0026	0.0032	-0.0014	0.0157
Cadburys	28%	26%	-0.0508	1.0000	0.0565	0.1073	0.0264	-0.0006
Access Bank	10%	30%	-0.0026	0.0565	1.0000	0.0418	0.1658	0.0052
AIICO Insur.	35%	15%	-0.0032	0.1073	0.0418	1.0000	0.0267	0.0012
Zenith Bank	23%	34%	-0.0014	0.0264	0.1656	0.0267	1.0000	-0.0073
Nestle Foods	18%	36%	0.0157	-0.0006	0.0052	0.0012	-0.0073	1.0000

Table 2: Empirical correlation matrix of some assets considered in NSM with their assumed weights and Implied volatilities.

Thus, from the asset return from NSM the empirical correlation matrix

$$C^P = \begin{bmatrix} 1.0000 & -0.0508 & -0.0026 & 0.0032 & -0.0014 & 0.0157 \\ -0.0508 & 1.0000 & 0.0565 & 0.1073 & 0.0264 & -0.0006 \\ -0.0026 & 0.0565 & 1.0000 & 0.0418 & 0.1656 & 0.0052 \\ 0.0032 & 0.1073 & 0.0418 & 1.0000 & 0.0267 & 0.0012 \\ -0.0014 & 0.0264 & 0.1656 & 0.0267 & 1.0000 & -0.0073 \\ 0.0157 & -0.0006 & 0.0052 & 0.0012 & -0.0073 & 1.0000 \end{bmatrix}$$

The eigenvalues of $C^P = [1.2205,0.8308,0.8808,1.0675,0.9843,1.0148]'$. Thus the minimum eigenvalue of $C^P = 0.8308$ which shows that C^P is a valid correlation matrix. Therefore, to estimate the realistic implied correlation matrix C^Q from the assumed implied volatility for the given portfolio consisting of six assets, we assume that the implied volatility of portfolio $\sigma^Q_{port} = 0.28$.

From Table 1: The weights of the respective assets $W = [0.15\ 0.28\ 0.10\ 0.35\ 0.23\ 0.18]$;

$$S^Q = \begin{bmatrix} 0.23 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.26 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.30 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.15 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.34 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.36 \end{bmatrix}$$

We now use equation (19) and the respective values of W, S^Q, C^P, to calculate σ^P_{port}.

$$\sigma^P_{port} = SQRT(W * S^Q * C^P * S^Q * W') = 0.1546.$$

Since $0.28 > 0.15 \rightarrow \sigma^Q_{port} > \sigma^P_{port}$, therefore we shall replace C in equation (22) by an equivalent identity 6x6 equicorrelation matrix to obtain the value of w:

$$w = \frac{(\sigma^Q_{port})^2 - (\sigma^P_{port})^2}{W * S^Q * (I_{6x6} - C^P) * S^Q * W'} = \frac{0.0545}{W * S^Q * (I_{6x6} - C^P) * S^Q * W'}$$

$= \frac{0.0545}{0.0914} = 0.5963$

Therefore, $C^Q = C^P + 0.5963 * (I_{6x6} - C^P)$

$$= \begin{bmatrix} 1.000 & 0.576 & 0.595 & 0.598 & 0.596 & 0.603 \\ 0.576 & 1.000 & 0.619 & 0.640 & 0.607 & 0.596 \\ 0.595 & 0..619 & 1.000 & 0.613 & 0.663 & 0.598 \\ 0.598 & 0.640 & 0.613 & 1.000 & 0.607 & 0.597 \\ 0.596 & 0.607 & 0.663 & 0.607 & 1.000 & 0.593 \\ 0.603 & 0.596 & 0.598 & 0.597 & 0.593 & 1.000 \end{bmatrix}$$

The eigenvalues of $C^Q = [4.034, 0.445, 0.430, 0.398, 0.357, 0.336]'$ from where we obtain the minimum eigenvalue to be 0.3359 showing that C^Q is also positive semi-definite. We now verify our solution from equation (16) to compute the variance of portfolio using the obtained realistic implied correlation matrix C^Q gotten above:

$$\sigma^P_{port} = SQRT\ (W * S^Q * C^Q * S^Q * W' = 0.28 \text{ as required.}$$

Contribution to Knowledge

The research has provided an insight into the dynamics of bank assets price correlation in the Nigerian Stock Market and consequently the information on the best risk management practices for bank investors in the Exchange. The empirical correlation matrix so obtained has shown that most of the bank stocks of NSM move in the same direction except the Union bank and Unity banks that have inverse correlation with the other banks. For an investor in the NSM it therefore pays to have stakes in other non-bank stocks if he wants to diversify his portfolio in the market. It is, therefore, advisable to include derivative asset products due for introduction in the NSM to hedge against risk associated with having stocks in the banking sector especially when the stock prices of bank assets go down. The implied correlation matrix is applicable in hedging risks associated with foreign exchange. Large corporations are always very interested in hedging their currency exposures by using a basket of options instead of taking separate put options for the respective countries where they have their investments [57]. This will help guard against the unnecessary losses they might incur in an event of rising value of the domestic currency where they have these investments. Companies that are therefore exposed to a variety of currency fluctuations find it profitable to directly hedge their aggregate risk by using a basket of options made possible through the use of estimated implied correlation matrix from a basket of options [58]. For a manufacturing firm in the United States that sources its raw materials in Nigeria, Ghana and South Africa and pays for its operation in those countries in local currencies is likely to be exposed to exchange rate risk. To hedge against the risk or falling prices of the United States dollars against Naira, Cedi and Rand, the manufacturing firm has to use a basket of option in its risk management strategy. The company could therefore directly buy an option on a basket of currencies at a lower price than it can purchase through separate options on the individual currencies. This is possible through the use of historical return time series correlation results, as we have done with some stocks in the NSM, Urama T.C.et al; [39], and the major concern will then be the amount of weight to be assigned to the individual stocks (or currencies). For optimal portfolio on the investment, we need to predict correctly the future correlation of the respective option values by observing the correlation throughout the life span of the option.

Appendix 1

DERIVATION OF MARCENKO-PASTUR LAW (DISTRIBUTION)

The Marcenko-Pastur (M-P) law investigates the level density for various ensembles of positive matrices of a Wishart-like structure which is denoted by $W = XX^T$, where X stands for a random matrix. In particular, for some stocks in the Nigerian Stock Market (NSM), we have $R = \frac{1}{L}X^T X$ with L as the period of time considered in the time series and we make use of the Cauchy transform to derive the M-P distribution.
To derive the level density associated with a given ensembles of random matrices, and in a more general sense some free convolutions of the M-P law, we will use the Voicucescu S-transform and the Cauchy functions.
Suppose that $X = (X_1, X_2, \ldots, X_n) \in \mathbb{R}^{pxl}$ where X_i, are independent and identically distributed with mean zero and variance one. Furthermore, let's define

$$\mathbb{R}_n = \frac{1}{L}XX^T \in \mathbb{R}^{pxp} \quad (1)$$

and let $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ denote the eigenvalues of the matrix $\mathbb{R}_n$. In particular, from the data used in my research for the stock from Nigerian Stock Market, $L = 3101, P = 15$. Suppose we define the random spectral measure by

$$\mu_n = \frac{1}{n}\sum_{i=1}^{n} f(\lambda_i) \quad (2)$$

where $\lambda_{i's}$ are the eigenvalues of the random matrix, we can then state the, M-P distribution as follows:

Marcenko-Pastur Law (Distribution): If $\mathbb{R}_n$, μ_n are defined as in (1) and (2) above, and suppose further that $^{p}/_{L}$ approaches $Q \in (0,1)$ where p, L are sufficiently large, then we have $\mu_n(., w) \Rightarrow \mu$ almost surely (a.s) with µ known to have a deterministic measure whose density is given by

$$\wp(\lambda) = \frac{d\mu}{dx} = \frac{Q}{2\pi x}\sqrt{(b-x)(x-a)}|_{a \leq x \leq b} \quad (3)$$

Here, a and b are functions of Q given by $a(Q) = (1-\sqrt{Q})^2, b(Q) = (1+\sqrt{Q})^2$ with a and b representing λ_{min} *and* λ_{max} respectively in the thesis.
Remark: We observe that when the rectangular parameter $Q = 1, with\ a = 0, b = 4$ we shall have

$$\wp(\lambda) = \frac{d\mu}{dx} = \frac{1}{2\pi x}\sqrt{(4-x)x}|_{0 \leq x \leq 4} \equiv \frac{1}{2\pi}\sqrt{\frac{4-x}{x}} \quad (4)$$

which yields the image of a semicircle distribution under the mapping $x \to x^2$.
The variable x represents a suitably rescaled eigenvalue λ of $\mathbb{R}_n$. For normalized random Wishart matrix, with respect to the trace condition $Tr\ \mathbb{R}_n = 1$, the rescaled variable is $x = \lambda N$, where N is the size of matrix, X.
We now use the S-transform that corresponds to an unknown probability measure defined on a complex variable ω, on the x-axis for the analysis of M-P distribution defined as

$$S_{M-P}(\omega) = \frac{1}{1+\omega} \quad (5)$$

To infer this measure and the spectral density $\wp(\lambda)$, Mlotkowski et al. (2015) write the S-transform as

$$S(\omega) = \frac{1+\omega}{\omega}\chi(\omega) \quad (6)$$

where,
$$\frac{1}{\chi(\omega)} G\left\{\frac{1}{\chi(\omega)}\right\} - 1 = \omega \quad (7)$$

Suppose we set the characteristics function χ(ω) as

$$\frac{1}{\chi(\omega)} = z\ with\ z \in ₵^+ \equiv \{z \in ₵ : Im\ z > 0\} \quad (8)$$

This will enable us to obtain the implicit solution to the Green's function G(z) which can also be referred to as the Cauchy function written as:

$$G(z) = \frac{1}{n}\ tr(A - zI)^{-1} \quad (9)$$

where, A connotes a random matrix from the ensemble investigated (which in this work represents the 82 stocks considered drawn from market prices in the Nigerian Stock Market).
Putting equation (8) into (7) we shall have:

$$zG(z) - 1 = \omega \Rightarrow zG(z) = 1 + \omega\ or\ G(z) = \frac{1+\omega}{\omega}$$

Thus from (9) above

$$G(z) = \frac{1}{n}\, tr\,(A - zI)^{-1} = \frac{1+\omega}{\omega} \tag{10}$$

Furthermore, from (6) and (8) we shall have: $s(\omega) = \frac{1+\omega}{z\omega}$ which implies that

$$zwS\big(\omega(z)\big) = 1 + \omega(z) \tag{11}$$

We now demonstrate how to obtain the general form of the M-P distribution which describes the asymptotic level density $\wp(\lambda)$ of random states of $\wp = \frac{XX^T}{TrXX^T}$, where X is the rectangular complex Ginibre matrix of size NxM, with the chosen rectangular parameter $Q = M/N \leq 1$.

Consider another S-transform similar to that of equation (5) defined as:

$$S_c(\omega) = \frac{1}{1+c\omega} \tag{12}$$

which reduces to equation (5) for $c = 1$ and putting equation (12) into (11) we shall obtain:

$z\omega(z)\left(\frac{1}{1+c\omega}\right) = 1 + \omega(z)orz\omega = (1+\omega)(1+c\omega)$

$$\Rightarrow c\omega^2 + \omega(c+1-z) + 1 = 0 \tag{13}$$

By solving the quadratic equation in terms of ω using the general formula we shall obtain:

$$\omega = \frac{-(c+1-z) \pm \sqrt{(c+1-z)^2 - 4c}}{2c}$$

$$= \frac{-(c+1-z) \pm \sqrt{(c^2 - 2c(1+z) + (1-z)^2}}{2c}$$

$$= \frac{-(c+1-z) \pm \sqrt{(c-1-\sqrt{z})^2 - (c-1+\sqrt{z})^2}}{2c}$$

Thus, the imaginary part of ω is zero when c lies outside the interval$[(1-\sqrt{z})^2, (1+\sqrt{z})^2]$. Finally, to obtain the spectral density as shown in equation (3), we apply the Stieltjes inversion formula and since the negative imaginary part of the Green's function yields the spectral function,

$$\wp(\lambda) = -\frac{1}{\pi}\lim_{\varepsilon\to 0} \mathcal{I}mG(z)|_{z=\lambda+i\varepsilon} \tag{14}$$

we shall have:

$$\wp(\lambda) = \frac{d\mu}{dx} = \frac{Q}{2\pi x}\sqrt{(b-x)(x-a)}|_{a\leq x\leq b}, \text{ as required;}$$

where $a(Q) = (1-\sqrt{z})^2 \equiv (1-\sqrt{Q})^2$ and $b(Q) = (1+\sqrt{z})^2 \equiv (1+\sqrt{Q})^2$and x being a dummy variable is represented by c.

The M-P equation has undergone several reformulations since its first appearance in the original paper of Marcenko-Pastur (1967). Some of this reformulation process was the instantiation of the equation by Silverstein and Bai under four different assumptions for the derivation of the theorem. To this end therefore, one derives the Marcenko-Pastur law as above using Marcenko-Pastur Theorem or the Silverstein and Bai Theorem (1995) as stated below:

Consider an NxN matrix, B_N . Assume that

(a) $X_n$is annxN matrix such that the matrix elementsX_{ij}^n are independent identically distributed (i.i.d) complex variables with mean zero and variance 1,$i.e\ X_{ij}^n \in ₵, E\big(X_{ij}^n\big) = 0\ \&\ E\big(||X_{ij}^n||^2\big) = 1$

(b)$p = n(N) with\ {}^{n}/_{N} \to c > 0\ as\ N \to \infty$. In particular, for the Nigerian bank stocks that were considered, $n = 15, N = 3101 \ni {}^{n}/_{N} = {}^{15}/_{3101} = 0.0048 > 0$.

(c) $T_n = diag(\tau_1^n, \tau_2^n, \ldots, \tau_n^n)$where$\tau_i^n \in \mathbb{R}$ and the eigenvalue distribution function (e.d.f) of $\{\tau_1^n, \tau_2^n, \ldots, \tau_n^n\}$ converges almost surely in distribution to a probability distribution function (p.d.f) $H(\tau) as\ N \to \infty >$

(d) $B_N = A_N + \frac{1}{N}X_n^* T_n X_n$, where $A_N$is a Hermitian NxN matrix for which F^{A_N} converges vaguely to Æ almost surely, Æ being a possibly defective (i.e with discontinuities) nonrandom distribution function

(e) $X_n, T_n\ and\ A_n$are independent.

Then, almost surely, F^{B_N}converges vaguely, almost surely, as $N \to \infty$ to a non-random distribution function (d.f) F^B whose Stieltjes transform $m(z), z \in ₵$ satisfies the canonical equation

$$m(z) = m_A\left(z - c\int \frac{\tau dH(\tau)}{1+\tau m(z)}\right) \tag{15}$$

We begin by defining the Stieltjes transform in an eigenvalue distribution which has proven to be an efficient tool for determining a limiting density. For every non-real z, the Stieltjes (or Cauchy) transform of the probability measure $F^A(x) = F[A(x)](z)$ is given by

$$m_A(z) = \int_{-\infty}^{\infty} \frac{1}{x-z} dF^A(x) \quad (16)$$

with$z = x + iy \in ₵, y \neq 0.$

Suppose $A_N = 0,$ from (d) above, $B_N = \frac{1}{N} X_n^* T_n X_n$. The Stieltjes transform of A_N, from definition (16) above will then be

$$m_A(z) = \frac{1}{0-z} = -\frac{1}{z}$$

and using Marcenko-Pastur theorem as expressed in equation (15) above, the Stieltjes transform $m(z)$ of B_N is given by

$$m(z) = -\frac{1}{z - c\int \frac{\tau dH(\tau)}{1+\tau m(z)}} \quad (17)$$

we can therefore find that the inverse of $m(z)$ will be given by

$$z = -\frac{1}{m} + c\int \frac{\tau}{1+\tau m} dH(\tau) \quad (18)$$

Equation (18) can be seen as an expression of relationship between the Stieltjes transform variable m and the probability space z which can alternatively be referred to as a canonical equation or functional inverse of $m(z)$.

Thus, to determine the density ofB_N as defined in (d) above using inversion formula (14) we need to solve (18) for $m(z)$. Hence, to be able to simplify the relationship between m and z we need to obtain $dH(\tau)$ from equation (18). Theoretically, $dH(\tau)$ could be regarded as any density which satisfies the conditions of Marcenko-Pastur theorem. In Particular, for some specific distribution of $dH(\tau)$ we can obtain the density analytically.

For $T_n = 1,$ in (c) above of the theorem, which coincidentally is the same as was observed from the empirical matrix (as the diagonal elements of T_n are non-random) with distribution function). We note here that for general forms of the probability distribution $H(\tau)$ it is not possible to find an analytic solution for m in (18) above, however, for the well-known white Parcenko -Pastur or canonical form of the distribution, equation (18) can be solved using the relation$dH(\tau) = \delta(\tau - 1)$ to obtain $z = -\frac{1}{m} + \frac{c}{1+m}$. Thus, with $T_n = 1$ we obtain from equation (18)

$$z = -\frac{1}{m} + \frac{c}{1+m}$$

$$\Rightarrow z(m)(1+m) = -(1+m) + cm \qquad \text{or } m^2 z + m(1 - c + z) + 1 = 0 \quad (20)$$

which is analogous to the expression represented by equation (13) and solving as before we can therefore obtain the solutions of m in terms of z.

Thus, to obtain the density which is usually referred to as the Marcenko-Pastur distribution we solve the quadratic equation in (20) above and make use of equation (14) to get:

$$\wp(\lambda) = \frac{dF^B(x)}{dx} = \frac{d\mu}{dx} = \frac{Q}{2\pi x}\sqrt{(b-x)(x-a)}\Big|_{a \le x \le b}, \text{ as obtained before.}$$

References

[1] Mehta, M.I. (2004), Random Matrices, Elsevier 3rd Edition Amsterdam

[2] I. Molero-Gonzalez et al. (2023). Market Beta is not dead: An Approach from Random Matrix Theory, Finance Research Letters 55, 1-7; Elsevier

[3] AlifianMahardhika and AcepPurgon (2016). Random Matrix Theory Approach to Indonesia Energy Portfolio Analysis; International Conference on Energy Sciences (ICES 2016), Conference series, Journal of Physics: Conf. series 877 (2017) , 1-6; 012022, JOP Publishing

[4] Szilard Pafka and ImreKondor (2004).Estimated correlation matrices and portfolio optimization, Physica A, 343, 623-634.

[5] KaweeNumpacharoen (2013). Weighted average correlation matrices method for correlation stress testing and

sensitivity analysis; The Journal of Derivatives, 67-74.
[6] A. Sensoy, S. Yuksel, M. Erturk (2013).Analysis of cross-correlations between financial markets after the 2008 crisis; Physica A, 392, 5027-5045.
[7] R.N. Mantegna, Hierarchical structure in financial markets (1999). The European Physical Journal B, 11, 193-197.
[8] L. Laloux, P. Cizeau, J. Bouchaud, M. Potters (1999).Noise dressing of financial correlation matrices, Physical Review Letters, 83, 1467-1470.
[9] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, H.E. Stanley (1999).Universal and nonuniversal properties of cross-correlations in financial time series; Physical Review Letters, 83, 1471-1474.
[10] Jae Dong Noh (2000).A model for correlations in stock markets; Physical Review E, 61(5), 1-2.
[11] S. Sharifi, M. Crane, A. Shamaie, H. Ruskin (2004). Random matrix theory for portfolio optimisation: a stability approach; Physica A, 335, 629-643.
[12] Ivailo I. Daimov, P.N.Kolm, L. Maclin, D.Y.C. Shiber (2012). Hidden noise structure and random matrix models of stock correlations; Quantitative Finance, 567-572.
[13] Bernd Rosenow, ParameswaranGopkrishnan, VasilikiPlerou, H. Eugene Stanley (2003).Dynamics of cross-correlations in the stock market, Physica A, 324, 241-246.
[14] Stanislaw Drozdz, Frank Grummer, J. Speth (2001). Towards identifying the world stock market cross-correlations: DAX versus Dow Jones, Physica A: Statistical Mechanics and its Applications, 226-234.
[15] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, H.E. Stanley (2000). A random matrix theory approach to financial cross-correlations; Physica A, 287, 374-382.
[16] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, H.E. Stanley(2001). Collective behaviour of stock price movements: A random matrix theory approach; Physica A, 299, 175-180.
[17] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, T. Guhr, H.E. Stanley (2002).Random matrix approach to cross-correlations in financial data; Physical Review E 65, 066 -072.
[18] B. Rosenow, V. Plerou, P. Gopikrishnan (2000).Application of random matrix theory to study cross-correlations of stock prices; International Journal of Theoretical and Applied Finance, 3(3), 399-403.
[19] G. Wang, C.Xie, S. Chen, J. Yang, M. Yang (2013). Random matrix theory analysis of cross-correlations in the US stock market: Evidence from Pearson's correlation coefficient and detrended cross-correlation coefficient, Physica A; 392, 3715-3730.
[20] Juan Martin Gonzalez and Antonio Alatorre Torres (2015).Random matrix approach to correlation matrix of financial data (Mexican Stock Market case); Modern Economy, 6, 1033-1042. http://www.scirp.org/journal/mc 2015.9099, scientific research publishing.
[21] Marwane El- Alaoui (2015).Random matrix theory and portfolio optimization in Mexican Stock Exchange; Physica A, 92-99.
[22] Varsha Kulkarni and NiveditaDeo (2007). Correlation and volatility in an Indian Stock Market: A random matrix approach, European Physical Journal B, 60, 101-109.
[23] R. Kumar and S. Sinha, (2007).Collective behaviour of stock price movements in an emerging market; Physical Review E 76, 046-056.
[24] Feng Ma, Yu Wei and Dengshi Huang (2015). Multifractal detrended cross-correlations analysis between the Chinese Stock Market and surrounding stock markets, Physica A, 392, 1659-1670.
[25] Marc Potters, Jean-phillipeBouchaud, laurentLaloux (2005). Financial applications of random matrix theory: Old laces and new pieces; ACTA PhysicaPolonica B, 36(9), 2767-2783.
[26] J. Shen and B. Zheng (2009). Cross-correlation in financial dynamics, A Letters Journal Exploring the Frontiers of Physics, EPL 86, 48- 55
[27] Bernd Rosenow, ParameswaranGopkrishnan, VasilikiPlerou, H. Eugene Stanley(2002).Random magnets and correlations of stock price fluctuations; Physica A, 314, 762-767.
[28] S. Cukur, M. Eryigit, R. Eryigit (2007).Cross-correlation in an emerging market financial data; Physica A, 376, 555-564.
[29] L. Junior and I. Franca (2012).Correlation of financial markets in times of crisis; Physica A, 391, 187-208.
[30] Chandradewsharma and Kinjal Banerjee (2015). A study of correlations in the stock market; Physica A, 432, 321-330.
[31] Diane Wilcox and Tim Gebbie (2007).An Analysis of cross-correlations in an emerging market, Physica A, 375,584-598.
[32] M. Kim, S. Lee, D. Hwang, S. Kim, I. Ko (2007). Dynamics of implied volatility surfaces from random matrix theory, Physica A, 389, 2762-2769.

[33] A. Nobi, S. Maeng, G. Gyun, J. Lee (2013). Random matrix theory and cross-correlations in global financial indices and local stock market indices; Journal of the Korean Physical Society, 62(4), 569-574.
[34] L. Laloux, P. Cizeau, M. Potters J. Bouchaud (2000). Random matrix theory and financial correlations; International Journal of Theoretical and Applied Finance, 3, 1-7.
[35] P. Gopikrishnan, B. Rosenow, V. Plerou, E. Stanley (2001). Quantifying and interpreting collective behaviour in financial markets; Physical Review E, 64, 35 - 46.
[36] H. Markowitz (1952). Portfolio Selection; Journal of Finance, 2, 77-91.
[37] R. Engle and S. Figlewski, Modelling the dynamics of correlation among implied volatilities; Review Finance, 1-28.
[38] Fisher Black and Myron Scholes (1973).The pricing of options corporate liabilities; Journal of Political Economy, 81(3), 637-654.
[39] Urama, T.C., Nnanwa, C.P., & Ezepue, P.O. **(2017),** Analysis of Cross-Correlations in Emerging Markets using Random Matrix Theory; Journal of Mathematical Finance, 7, 291-301; http://www.scirp.orgjournal/jmf
[40] A. Edelman (1988).Eigenvalues and condition numbers of random matrices, SIAM Journal on matrix analysis and applications 9(4), 543-560.
[41] Gee-rae Kim and Harry, M. Markowitz (1989).Investment rules, margin and market volatility; The Journal of Portfolio Management, 16(1), 45-52.
[42] R.G. Palmer, W.B. Arthur, J.H Holland, B. LeBaron (1994). Artificial economic life: A simple model of a stock market; Physica D, 75, 264-274.
[43] I. Kondor and J. Kertesz, EconoPhysics (1999).An emerging Science, Kluwer, Dordrecht, 456 -472.
[44] A. Charterjee and B.k. Chakrabarti (2006).Econophysics of Stock and other markets, Springer 213 - 229.
[45] J. Voit (2001).The Statistical Mechanics of Financial Markets; Springer, Berlin 2001.
[46] H. Markowitz, Portfolio Selection (1959).Efficient Diversification of Investment, John Wiley and Sons, New York, 1959.
[47] Szilard Pafka and ImreKondor (2003).Noisy covariance matrices and portfolio optimization 2; Physica A, 319, 487-494.
[48] S. Galluccio, J. Bouchaud, M. Potters (1998). Rational decisions, random matrices and spin glasses; PhysicaA 259, 449-456.
[49] T. Colon, H.J. Ruskin, M. Crane (2007).Random Matrix Theory and fund of funds portfolio optimization; Physica A, 382, 565-576.
[50] F.J. Dyson, Rev. Mex. Fis 20, 231 (1971).
[51] A.M. Sengupta and P.P. Mitra (1999).Distributions of singular values for some random matrices; Physical Review E, 60(3), 1-4.
[52] Z.D. Bai (1999).Methodologies in spectral analysis of large dimensional random matrices, A Review, StatisticaSinica 9, 611-677.
[53] T. Guhr, A. Muller, H. Weidenmuller (1998). Random matrix theories in Quantum Physics: Common concepts; Physics Reports 299, 189-425.
[54] A. Utsugi, K. Ino, M. Oshikawa, Random matrix theory analysis of cross-correlations in financial markets, Physical Review E 70 (2004) 026110.
[55] KaweeNumpacharoen and NattachaiNumpacharoen(2013). Estimating realistic implied correlation matrix from option prices; Journal of Mathematical Finance 3, 401-406; http://www.scirp.org/journal/jmf
[56] Adrian Buss and GrigoryVilkov, Measuring equity risk with option-implied correlations; the review of Financial Studies, 25(10), 3113-3140.
[57] Hari Krishnan and Izzy Nelken (2001). Estimating implied correlations for currency basket of options using the maximum entropy method; Derivative use, Trading & Regulation,7(3), 275-283.
[58] J. Hull, Options, (1997). Futures and other Derivative Securities, Prince Hall, Englewood Cliffs, NJ. 1997.

Note:

The researcher is grateful to Tertiary Education Trust Fund Nigeria for funding this research through an Institution Based Research (IBR) sponsorship for Institute of Management and Technology (IMT) Enugu Nigeria.