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Black-Scholes Model and Profit and Loss Attribution of Options

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Abstract

In this paper, we will provide a reformulation of the Black-Scholes formula. Under the Black-Scholes assumptions, the gain and loss from the delta hedge is the change in the value of the option. In practice, the identity will serve as a theoretical foundation of the profit and loss attribution of the option throughout the life of the trade.

1.1 Introduction

European options are priced with Black-Scholes formula [2]. A modern treatment of the Black-Scholes model using Ito's calculus [10] can be found in [5]. The article [6] is another comprehensive exposition of the Black-Scholes model. In [15], the two derivations of the Black-Scholes model, the original bond replication approach by Black and Scholes, and the call replication approach by Merton are compared. The Black-Scholes equation has wide range of applications, for example, it is used to price weather derivatives in [11].

Generally, however, the constant volatility assumption will not hold. In practice, the volatility surface is introduced so that Black-Scholes formula can be used to reproduce the market price [9]. The volatility surface has usually two dimensions, maturity and strike level. The focus of the paper is to understand the theoretical foundation of the option trading activities in practice, to understand the profit and loss attribution of option trades and its associated hedge through the life of the trade.

The oil producers, airlines, insurers and others buy options from or sell options to investment banks to manage their risk from business activities. The investment bank on the other side of the trade meets those client demands, and will need to manage their own risk. One common approach used to manage such risk by investment bank is to delta hedge the options. With the delta neutral portfolio, option plus the hedging underlying, the investment bank is risk flat with respect to the movements of the underlying. For example, if the bank is long an out of money call option, if the underlying moves sideways throughout the term of the option, then the bank has a risk that the option expires without payoff and loses all the premium. However, in practice, as the bank delta hedges the option, it should expect some gains from the hedging activity. In this paper, we will show that the gain from the hedging activity will equal the price of the option. While this is a well-known fact, the mathematical derivation under the assumption of the flat and sideways movement of the market is new. The mathematical derivation presented in this paper is a reformulation of the Black-Scholes formula.

One of the practical implications of the somewhat theoretical re-formulation of the Black-Scholes formula, is the profit and loss attribution of the option trades. Profit and loss attribution is a critical subject with attention from academicians and market practitioners. The concept of the P&L attribution is based on the Taylor expansion

$$\frac{\Delta H_t}{\Delta t} = \frac{\partial H_t}{\partial t}\Delta t + \frac{\partial H_t}{\partial S_t}\Delta S_t + \frac{\partial H_t}{\partial \sigma}\Delta\sigma + \frac{1}{2}\frac{\partial^2 H_t}{\partial S_t^2}(\Delta S_t)^2 + \frac{\partial^2 H_t}{\partial S_t \partial\sigma}(\Delta S_t)(\Delta\sigma) + \frac{1}{2}\frac{\partial^2 H_t}{\partial \sigma^2}(\Delta\sigma)^2$$

Where $H_t = H(t,T,S_t,K,\sigma)$ is the option price at the time t, of the option with maturity T, underlying price S_t, strike K, and volatility σ. The partial derivative ($\frac{\partial H_t}{\partial t}$, theta) is effect of time decay, ($\frac{\partial H_t}{\partial S_t}$, delta) is the effect of the price movement, and ($\frac{\partial H_t}{\partial \sigma}$, vega) is the effect of the volatility movement. Others are second-order effects ($\frac{\partial^2 H_t}{\partial S_t^2}$, gamma; $\frac{\partial^2 H_t}{\partial S_t \partial\sigma}$, vanna; $\frac{\partial^2 H_t}{\partial \sigma^2}$, volga).

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Fundamental Review of the Trading Book (FRTB), which sets the new capital standards for market risk for investment banks, emphasizes the role of P&L attribution. It requires banks to perform the P&L attribution test; see [14]. The failure probabilities of the P&L Attribution (PLA) test were analyzed in [7]. In [13], the statistical analysis of the FRTB P&L attribution test is performed. It points out potential revisions of the test to make it more effective. In particular, the revisions are on the conceptual treatment of the hedged portfolio. Note that, in this paper, we are considering the underlying hedge positions together with the option/derivative positions as well.

As an example, see [3] for one approach for fixed income performance attribution, and [16] for well-known research on performance attribution of derivatives. The central question in [4] is the linkage between the pricing formula and the P&L attribution. In some sense, this paper is another approach of the central questions in [4] under a much-simplified assumption. This paper assumes the constant volatility under the classical Black-Scholes assumption. In contrast, in [4], a full volatility term structure is considered. As our assumption is much simpler, the conclusion in this paper is more "closed-form". The P&L attribution identity is obtained under the sideways market in continuous time, showing that the price is the same as the P&L attribution of the delta effect and the theta effect, (volatility is assumed to be constant). In [8], the hedging activity is considered together with the option in performance analysis as well.

The profit and loss attribution in option trades tries to allocate the change in option value to the effect of the market moving (change in underlying price S), the time value of the option (theta effect, change in time to maturity), and to the effect of the interest rate, and to the dividend etc. When the market moves sideways, the call option loses value through the theta effect; the gain, which will offset the loss, comes from the hedging activities. In fact, the hedging activity, for the long call option, is just buy low and sell high.

P&L attribution is also used to measure the performance/quality of a model. A superior model better explains the changes in model price using changes in risk factors and corresponding sensitivities. The Black-Scholes model was found to be a better model than the Heston model in [12]. In [1], a new metric is introduced, a backward-looking criterion which the authors believe to be sounder. In fact, in this paper, we prove that under the continuous time setting, in the sideways market, the price change is exactly the same as the P&L attribution.

While the paper uses the long European call as an example, with the assumption that the market moves sideways, the concept can be expanded and provide insight to other options and other market conditions.

The paper has two parts. The first part is the re-formulation of the Black-Scholes formula to prove that the hedge cost of the call option is the same as the call option price in the sideways market. The second part is the simulation result of the P&L attribution of a call option with one year maturity throughout the trade under the sideways market.

1.2 The Baseline Model and Theoretical Formulation

In this section, we will present our baseline model and assumptions. The option position is a single European call option, with strike K, underlying $S = S(0)$, maturity T, and the risk-free rate $r = 0$. The volatility is σ, without dividend. To simplify the calculation, it is carried out in the risk neutral world.

1.2.1 The Discrete Case

We divide the time interval $(0, T)$ to N subintervals, each of length $e = T/N$. The delta hedging of the call option is carried out at discrete points in time, $t = i \cdot e$ and at $t = (i+.5) \cdot e$, $i = 0, 1, 2, ..., N-1$, meaning the hedging position is $(-D(t, S(t)))$ share of the underlying stock. The $D(t,S) = \phi(d_1)$ where ϕ is the CDF (Cumulative Distribution Function) up until d_1, $d_1 = \frac{\ln(S/K)+\frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$.

We will assume the market moves sideways. The underlying $S(t)$ remains S at all points $t = i \cdot e$, $i = 0, 1, 2, ..., N-1$. And the underlying $S(t)$ will move to $S - M$ at the middle of the internal at time $(i + .5) \cdot e$. We will have to assume a drop of S within an interval so that the realized volatility would be the same as the pricing volatility, σ.

The cash at time $t = 0$ is $D(0, S) \cdot S$ when shorting $D(0,S)$ shares of underlying.

The cash from the rebalancing at $t = .5 \cdot e$ is $(D(.5 \cdot e, S - M) - D(0,S)) \cdot (S - M)$.

The cash from the rebalancing at $t = 1 \cdot e$ is $(D(1 \cdot e, S) - D(.5 \cdot e, S - M)) \cdot S$.

We continue to rebalance at all time points and middle of the interval. The sum of hedging cost as a function of N, T, S, and K is

$$H(N,T,S,K) = -D(0,S)S$$
$$+ \{-D(.5e,S - M) + D(0,S)\}(S - M)$$

$$+\ \{-D(e,S) + D(.5e,S-M)\}S \quad (1)$$

...

$$+\ \{-D(T,S) + D(T-.5e,S-M)\}S$$

Condensing, we have

$$H(N,T,S,K) = M(\sum_{i=0}^{N-1} D(i\cdot e,S) - \sum_{i=0}^{N-1} D((i+.5)\cdot e, S-M)) \quad (2)$$

Note that the $D(T,S)$ =0, there are no more hedging position at time T. At time T, the hedging portfolio only contains the cash from the hedging activities as expressed in (2), $H(N,T,S,K)$.

1.2.2 Continuous time and integration

To calibrate M, so that the realized volatility is the same as the pricing volatility, we have

$$N\cdot((S-M)/S-1)^2 + N\cdot(S/(S-M)-1)^2 = \sigma^2 \quad (3)$$

Then,

$$\lim_{N\to\infty}(2NM^2)/(\sigma^2S^2) = 1$$
$$\lim_{N\to\infty} 2NM^2 = \sigma^2S^2 \quad (4)$$

Now our goal is simplifying $M(\sum_{i=0}^{N-1} D(i\cdot e,S) - \sum_{i=0}^{N-1} D((i+.5)\cdot e, S-M))$. For each i, we have

$$\begin{aligned} D(ie,S) - D((i+.5)e,S-M) &= \frac{\partial}{\partial t}D(ie,S)(.5e) + \frac{\partial}{\partial S}D(ie,S)M + O(e) \\ &= \frac{\partial}{\partial S}D(ie,S)M + O(M^2) \end{aligned} \quad (5)$$

Where $\frac{\partial}{\partial t}D(t,S)$ and $\frac{\partial}{\partial S}D(t,S)$ are partial derivatives.

Now adding up over all i, we have

$$\begin{aligned} M(\sum_{i=0}^{N-1} D(i\cdot e,S) - \sum_{i=0}^{N-1} D((i+.5)\cdot e,S-M)) &= M(\sum_{i=0}^{N-1} M\frac{\partial}{\partial S}D(ie,S)) \\ &= M^2(\sum_{i=0}^{N-1}\frac{\partial}{\partial S}D(ie,S)) \\ &= NM^2\cdot\frac{1}{N}(\sum_{i=0}^{N-1}\frac{\partial}{\partial S}D(ie,S)) \end{aligned} \quad (6)$$

Let $H(T,S,K)$ be the hedging cost in continuous time in the sideways market. Then,

$$H(T,S,K) = \lim_{N\to\infty} H(N,T,S,K) = \frac{S^2\sigma^2}{2}\int_0^T \frac{\partial}{\partial S}D(t,s)\,dt \quad (7)$$

Where (6) is the standard Riemann sum of the definite integral found in (7).

1.2.3 Proving Black-Scholes

In this section, we will show that the hedging cost $H(T,S,K)$ is the same as the Black-Scholes call option price. Let

$$d_1(x) = \frac{\ln(S/K)+\frac{\sigma^2}{2}x^2}{\sigma x} \qquad d_2(x) = \frac{\ln(S/K)-\frac{\sigma^2}{2}x^2}{\sigma x}.$$

Set $d_1 = d_1(\sqrt{T})$ and $d_2 = d_2(\sqrt{T})$. Then

$$\begin{aligned} H(T,S,K) &= \frac{S^2\sigma^2}{2}\int_0^T (\frac{\partial}{\partial S}D(t,S))\,dt \\ &= \frac{S^2\sigma^2}{2\sqrt{2\pi}}\int_0^T (\frac{\partial}{\partial S}(\int_{-\infty}^{d_1(\sqrt{T-t})} e^{-\frac{1}{2}x^2}\,dx))\,dt \end{aligned} \quad (8)$$

We now evaluate $\frac{\partial}{\partial S}(\int_{-\infty}^{d_1(\sqrt{T-t})} e^{-\frac{1}{2}x^2}\,dx)$ First define $F(z) = \int_{-\infty}^{z} e^{-\frac{1}{2}x^2}\,dx$.

$$\begin{aligned}
\frac{\partial}{\partial S}(\int_{-\infty}^{d_1(\sqrt{T-t})} e^{-\frac{1}{2}x^2}\,dx) &= \frac{\partial}{\partial S}F(d_1(\sqrt{T-t})) \\
&= \frac{\partial}{\partial S}[d_1(\sqrt{T-1})]\cdot F'(d_1(\sqrt{T-t})) \\
&= \frac{\partial}{\partial S}[\frac{\ln(S/K)+\frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}]\cdot e^{-\frac{1}{2}(d_1(\sqrt{T-t}))^2} \\
&= \frac{1}{S\sigma\sqrt{T-t}}\cdot e^{-\frac{1}{2}(d_1(\sqrt{T-t}))^2}
\end{aligned} \tag{9}$$

So (8) becomes,

$$\begin{aligned}
\frac{S^2\sigma^2}{2\sqrt{2\pi}}\int_0^T(\frac{\partial}{\partial S}(\int_{-\infty}^{d_1(\sqrt{T-t})} e^{-\frac{1}{2}x^2}\,dx))\,dt &= \frac{S^2\sigma^2}{2\sqrt{2\pi}}\int_0^T \frac{1}{S\sigma\sqrt{T-t}}\cdot e^{-\frac{1}{2}(d_1(\sqrt{T-t}))^2}\,dt \\
&= \frac{S\sigma}{2\sqrt{2\pi}}\int_0^T \frac{1}{\sqrt{T-t}}\cdot e^{-\frac{1}{2}(d_1(\sqrt{T-t}))^2}\,dt
\end{aligned} \tag{10}$$

Let $y = \sqrt{T-t}$, we have $\frac{dy}{dt} = \frac{-1}{2\sqrt{T-t}}$. Now,

$\frac{S}{2\sqrt{}}$

(11)

Let $x = d_1(y)$, we have $\frac{dx}{dy} = \frac{-d_2(y)}{y}$.

$$\begin{aligned}
\frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_1(y))^2}\,dy &= \frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot\frac{y}{-d_2(y)}\,dx \\
&= \frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot(\frac{1}{\sigma}+\frac{\sigma y + d_2(y)}{-\sigma d_2(y)})\,dx \\
&= \frac{S}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2} + \frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot\frac{\sigma y + d_2(y)}{-\sigma d_2(y)}\,dx \\
&= SN(d_1) + \frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot\frac{\sigma y + d_2(y)}{-\sigma d_2(y)}\,dx
\end{aligned} \tag{12}$$

We evaluate $\frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot\frac{\sigma y+d_2(y)}{-\sigma d_2(y)}\,dx$.

$$\begin{aligned}
\frac{S\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}(x)^2}\cdot\frac{\sigma y + d_2(y)}{-\sigma d_2(y)}\,dx &= \frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_1(y))^2}\cdot(\frac{\sigma y + d_2(y)}{-\sigma d_2(y)})(\frac{-d_2(y)}{y})\,dy \\
&= \frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_1(y))^2}e^{\frac{1}{2}(d_2(y))^2}e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy \\
&= \frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_1(y)^2-d_2(y)^2)}e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy \\
&= \frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(2\ln(S/K))}e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy \\
&= \frac{S\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{\ln(K/S)}e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy \\
&= \frac{K\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy
\end{aligned} \tag{13}$$

Let $z = d_2(y)$, we have $\frac{dz}{dy} = \frac{-d_1(y)}{y}$.

$$\frac{K\sigma}{\sqrt{2\pi}}\int_0^{\sqrt{T}} e^{-\frac{1}{2}(d_2(y))^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})\,dy = \frac{K\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_2} e^{-\frac{1}{2}(z)^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})(\frac{y}{-d_1(y)})\,dz \tag{14}$$

But we know $\frac{\sigma y + d_2(y)}{-\sigma d_1(y)} = -\frac{1}{\sigma}$, so:

$$\begin{aligned}\frac{K\sigma}{\sqrt{2\pi}}\int_{-\infty}^{d_2} e^{-\frac{1}{2}(z)^2}\cdot(\frac{\sigma y + d_2(y)}{\sigma y})(\frac{y}{-d_1(y)})\,dz &= -\frac{K}{\sqrt{2\pi}}\int_{-\infty}^{d_2} e^{-\frac{1}{2}(z))^2}\,dz \\ &= -KN(d_2)\end{aligned} \tag{15}$$

Which gives us desired
$H(T,S,K) = SN(d_1) - KN(d_2)$

1.3 The Discrete Model and Simulation

1.3.1 The Baseline Case

In this section, we will use an example to discuss the implication of the conclusion in section 1.2 in the profit and loss attribution, starting with long a call option and delta hedge it through the term of the trade. Let $K = 110$, and $S = 100$, $T = 1$, $\sigma = 20\%$, $r = 0$. Let $N = 125$, assuming 250 trading days in the year, and the market moves down by M one day and moves back up to 100 the next. The daily move $M = 1.26$ is calibrated to the realized annual volatility of 20%. Hedge of the long call option is rebalanced at end of the day to be delta flat.

The cost of the option is $4.29 using Black-Scholes formula. As the market is flat over the year, the option expires worthless at end of the year. The basic profit and loss attribution for the time period i, with respect to market move and theta, are

Theta-Attribution(i) = $C(i,S(i)) - C(i-1,S(i))$ (16)

(17)

Market-Attribution(i) = $C(i-1,S(i)) - C(i-1,S(i-1))$
And

Total-P&L(i) = $C(i,S(i)) + C(i-1,S(i-1))$ (18)

where$C(i,S(i))$ is the price of call option at time i, with underlying price of S.

The Price of option is $4.29. Delta at $t = 0$ is 0.35. Assuming the realized volatility to be 20%, the daily move should be $1.26 in the sideways market. The delta hedge, initial and rebalancing trades, will go short total of 3.43 shares at the average price of $98.74, and will go long the same amount in aggregate with average price of 100. The gain from the hedges is 3.43 · (100–98.74) = 4.32. The option expires worthless, a loss of ($4.29). Loss from the theta-attribution is ($4.08), and the loss from the market-attribution is (0.22). On the hedging trades, the P&L is 3.43 · (100–98.74) = 4.32. See figure 1 for a summary.

Long call option	Strike	110.00	Underlying Price	100.00	P&L from options	(4.29)
	Pricing volatility	0.20	Option Maturity	1.00		
	Delta at $t = 0$	0.35	Option price	4.29		
Modeling Market	# Of days	250.00	Daily move step	1.26		
	Realized volatility	0.20				
Delta Hedging	Day one hedge # of shares	(0.35)	Ending Delta	(0.00)	P&L from hedges	4.32
	Total short	(3.43)	Total long	3.43		
	Average price of short	100.00	Average price of long	98.74		
Portfolio	$t = 0$		$t = 1$		Attribution	
	Option	4.29	Option	0.00	Theta	(4.08)
	Cash	35.33	Cash	4.32	Market move option	(0.22)
	Stock	(35.33)	Stock	(0.00)	Market move hedge	4.32
	Total	4.29	Total	4.32	Total	0.02
			Gain	0.02	Check	-

Figure 1: Illustrative model for theta and market profit and loss attribution of long call option and its hedges.

1.3.2 Sensitivities to Realized Volatility

In the above case, we assumed the realized volatility is the same as the pricing vol. In practice, no one can predict what the realized volatility will be. When the realized volatility is higher than the pricing vol, the hedging cost (gain in reality) will be more than the price of the option. In our setting, the resulting cash would be more than the option price. As shown in Figure 3, the gain from hedge is $6.59, resulting in a gain of $2.30 in aggregate. On the other hand, if the realized volatility is only 15%, the gain from hedging is only $2.50, resulting in a loss of $1.79 in aggregate, see Figure 2.

Long call option	Strike	110.00	Underlying Price	100.00	P&L from options	(4.29)
	Pricing volatility	0.20	Option Maturity	1.00		
	Delta at $t = 0$	0.35	Option price	4.29		
Modeling Market	# Of days	250.00	Daily move step	0.95		
	Realized volatility	0.15				
Delta Hedging	Day one hedge # of shares	(0.35)	Ending Delta	(0.00)	P&L from hedges	4.32
	Total short	(2.64)	Total long	2.64		
	Average price of short	100.00	Average price of long	99.05		
Portfolio	$t = 0$		$t = 1$		Attribution	
	Option	4.29	Option	0.00	Theta	(4.13)
	Cash	35.33	Cash	4.32	Market move option	(0.16)
	Stock	(35.33)	Stock	(0.00)	Market move hedge	2.50

	Total	4.29	Total	2.50	Total	(1.79)
			Gain	(1.79)	Check	(0.00)

Figure 2: Profit and loss through the year when the realized volatility is 15%.

Long call option	Strike	110.00	Underlying Price	100.00	P&L from options	(4.29)
	Pricing volatility	0.20	Option Maturity	1.00		
	Delta at $t = 0$	0.35	Option price	4.29		
Modeling Market	# Of days	250.00	Daily move step	1.57		
	Realized volatility	0.25				
Delta Hedging	Day one hedge # of shares	(0.35)	Ending Delta	(0.00)	P&L from hedges	6.59
	Total short	(4.19)	Total long	4.19		
	Average price of short	100.00	Average price of long	98.43		
Portfolio	$t = 0$		$t = 1$		Attribution	
	Option	4.29	Option	0.00	Theta	(4.03)
	Cash	35.33	Cash	6.59	Market move option	(0.27)
	Stock	(35.33)	Stock	(0.00)	Market move hedge	6.59
	Total	4.29	Total	6.59	Total	2.30
			Gain	(2.30)	Check	(0.00)

Figure 3: Profit and loss through the year when the realized volatility is 25%.

1.4 Concluding Comments

In this paper, we definitively demonstrated the relationship between price and Greeks/P&L attribution for the Black-Scholes formula under the sideways market. This research shows that the price is the same as the sum of the P&L attributions with continuous time in the sideways market using some simple integral manipulation. The price and Greek relationship being the P&L relationship is the central topic in [4]. The result also provided certain theoretical insight for the conclusion in [1] that the Black-Scholes model is a "better" model than the Heston model in the sense that the P&L attribution explains the prices better. This research can be expanded to other models and other market conditions.

It is very simplifying to assume the market moves sideways and with a zero risk-free rate. The portfolio is also simple, being a delta hedge European call option. However, the formulation and analysis in this paper provide further insights to the profit and loss attributions of options trades. Both theta and market effects would be reflected in the profit and loss of the hedging activities.

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